

Math422 Ring Theory: Assignment 2, February 2011

Please show all working and reasoning to get full marks for any question. Please only discuss general ideas with your classmates, all work handed in should be your own and plagiarism will be punished. Attach all rough work attempted to show your thought processes.

1. Let R be $\mathbb{Z}_6[\sqrt{2}]$ and choose two non-trivial ideals of R different from everyone in the class; A which is not a principal ideal, if possible, and B which is a principal ideal. [3]
2. Determine the elements of R/A and R/B and give all details of a homomorphism from R onto a smaller ring using one of your two ideals. [3]
3. Verify that $A + B$ and $A \cap B$ are both subrings and ideals and explain if they are or are not principal ideals for you. [4]
4. Check the third isomorphism theorem for A and B by explaining the structures of $A/(A \cap B)$ and $(A + B)/B$. [4]
5. Explain why, for a general X and Y , $f(x) = x + Y$ is a homomorphism from X onto $(X + Y)/Y$ and show that the kernel of f is $X \cap Y$. Why does this prove the isomorphism theorem? [5]
6. Assume that S is a ring and C is an ideal of S . Prove that if S/C is a field then C is a maximal ideal by supposing the existence of an ideal D between C and S and getting a contradiction. [5]
7. Are either of your A and B maximal ideals? [1]