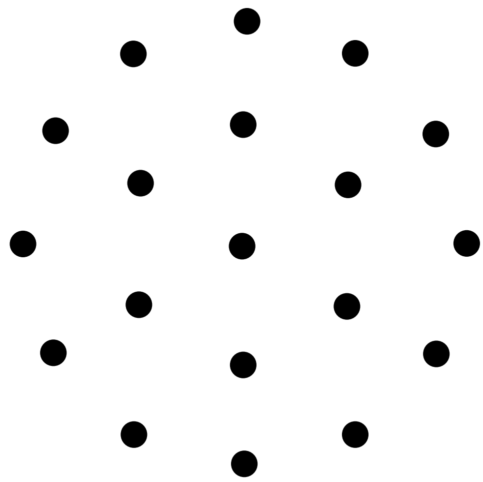
Math421 Group Theory: Assignment 4 April 2008

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

1. Suppose we have the arrangement shown in the figure:



The board can be rotated or flipped over on any of its axes and look the same. You should use Cauchy-Frobenius to count various different arrangements and not just give me the numbers involved, but the reasoning behind them:

   (a) How many different ways to put two identical colours on the vertices? [3]

   (b) How many different ways to put two different colours on the vertices? [3]

   (c) How many different ways to put one colour on any vertex and another on the any of the outside vertices? [3]

   (d) How many different ways to colour all vertices apart from those on the outside with n colours? What number do you get for $n = 2$? [3]

   (e) Depending on which assignment 3 you were allocated, choose the matching question from above and list logically all the arrangements you found. [4]

   (f) Again for only your one of the first four questions, generalise your count to the arrangement with 37 vertices (but don't list them). [3]

2. Suppose G is a group with 90 elements.

   (a) What are the sizes and the possible numbers of elements in the Sylow-p-subgroups of G? [2]

   (b) Explain why G must be non-simple if all non-identity elements in the Sylow-3-subgroups are distinct. [3]

   (c) Consider the only other possible intersection of two Sylow-3-subgroups and then the normaliser of that group to conclude that G cannot be simple. [6]