Math421 Group Theory: Assignment 3a March 2008

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

Throughout this question we will be working with the group G with this presentation:

$$G := \{x, y; x^4 = e, y^5 = e, yx = xy^2\}$$

1. (a) Use coset enumeration to find the number of elements of G. Use colours and/or multiple copies of your tables to show how you progressed. [5]

   (b) Draw the Cayley Diagram using x and y as your generators. Emphasize the symmetries in your group. Identify your cosets from (a). [3]

   (c) Identify all of the elements of G in terms of $y^i x^j$ and, if it uses fewer letters, the shortest path of arrows in the Diagram from the origin to the element. [3]

   (d) Using the group table or the Cayley Diagram get a representation of each of the elements as permutations as explained in class and check several non-trivial multiplications to be sure you are correct. [7]
   [use the letters A, B, C, ... to represent numbers above 9 if you like]

2. (a) Find the centre of G, and call it Z. [2]

   (b) Identify which group G/Z is isomorphic to. [2]

   (c) Now suppose H is any group such that $H/Z(H)$ is isomorphic to $C_n :=< a >$ for some integer n. Deduce the form of the cosets in terms of a and H. [2]

   (d) Choose any two elements from any two cosets and show they commute. What does this tell you about H? [4]

   (e) Suppose now that $|J| = p^2$ for some prime number p. Explain why there are only three possible orders for subgroups and if $|Z(J)| > 1$ then J must be one of just two specific groups. [3]

Math421 Group Theory: Assignment 3b March 2008

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

Throughout this question we will be working with the group G with this presentation:

$$G := \{x, y; x^4 = e, y^4 = e, xy = yx^3\}$$

1. (a) Use coset enumeration to find the number of elements of G. Use colours and/or multiple copies of your tables to show how you progressed. [5]

   (b) Draw the Cayley Diagram using x and y as your generators. Emphasize the symmetries in your group. Identify your cosets from (a). [3]

   (c) Identify all of the elements of G in terms of $y^i x^j$ and, if it uses fewer letters, the shortest path of arrows in the Diagram from the origin to the element. [3]

   (d) Using the group table or the Cayley Diagram get a representation of each of the elements as permutations as explained in class and check several non-trivial multiplications to be sure you are correct. [7]
   [use the letters A, B, C, ... to represent numbers above 9 if you like]

2. (a) Find the centre of G, and call it Z. [2]

   (b) Identify which group G/Z is isomorphic to. [2]

   (c) Now suppose H is any group such that $H/Z(H)$ is isomorphic to $C_n :=< a >$ for some integer n. Deduce the form of the cosets in terms of a and H. [2]

   (d) Choose any two elements from any two cosets and show they commute. What does this tell you about H? [4]

   (e) Suppose now that $|J| = p^2$ for some prime number p. Explain why there are only three possible orders for subgroups and if $|Z(J)| > 1$ then J must be one of just two specific groups. [3]

Math421 Group Theory: Assignment 3c March 2008

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

Throughout this question we will be working with the group G with this presentation:

$$G := \{x, y; x^3 = e, y^7 = e, yx = xy^2\}$$

1. (a) Use coset enumeration to find the number of elements of G. Use colours and/or multiple copies of your tables to show how you progressed. [5]

   (b) Draw the Cayley Diagram using x and y as your generators. Emphasize the symmetries in your group. Identify your cosets from (a). [3]

   (c) Identify all of the elements of G in terms of $y^i x^j$ and, if it uses fewer letters, the shortest path of arrows in the Diagram from the origin to the element. [3]

   (d) Using the group table or the Cayley Diagram get a representation of each of the elements as permutations as explained in class and check several non-trivial multiplications to be sure you are correct. [7]
   [use the letters A, B, C, ... to represent numbers above 9 if you like]

2. (a) Find the centre of G, and call it Z. [2]

   (b) Identify which group G/Z is isomorphic to. [2]

   (c) Now suppose H is any group such that $H/Z(H)$ is isomorphic to $C_n :=< a >$ for some integer n. Deduce the form of the cosets in terms of a and H. [2]

   (d) Choose any two elements from any two cosets and show they commute. What does this tell you about H? [4]

   (e) Suppose now that $|J| = p^2$ for some prime number p. Explain why there are only three possible orders for subgroups and if $|Z(J)| > 1$ then J must be one of just two specific groups. [3]

Math421 Group Theory: Assignment 3d March 2008

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

Throughout this question we will be working with the group G with this presentation:

$$G := \{x, y; x^4 = e, y^3 = e, xy = y^2x\}$$

1. (a) Use coset enumeration to find the number of elements of G. Use colours and/or multiple copies of your tables to show how you progressed. [5]
   (b) Draw the Cayley Diagram using x and y as your generators. Emphasize the symmetries in your group. Identify your cosets from (a). [3]
   (c) Identify all of the elements of G in terms of $y^i x^j$ and, if it uses fewer letters, the shortest path of arrows in the Diagram from the origin to the element. [3]
   (d) Using the group table or the Cayley Diagram get a representation of each of the elements as permutations as explained in class and check several non-trivial multiplications to be sure you are correct. [7]
   [use the letters A, B, C, ... to represent numbers above 9 if you like]

2. (a) Find the centre of G, and call it Z. [2]
   (b) Identify which group G/Z is isomorphic to. [2]
   (c) Now suppose H is any group such that $H/Z(H)$ is isomorphic to $C_n :=< a >$ for some integer n. Deduce the form of the cosets in terms of a and H. [2]
   (d) Choose any two elements from any two cosets and show they commute. What does this tell you about H? [4]
   (e) Suppose now that $|J| = p^2$ for some prime number p. Explain why there are only three possible orders for subgroups and if $|Z(J)| > 1$ then J must be one of just two specific groups. [3]