Math421 Group Theory: Assignment 2a February 2008

Please show all working and reasoning to get full marks for any question.

1. (a) List all of the different integer factorisations of 23, 27 and 40 and hence use the Fundamental Theorem of Abelian Groups to make all of the possible Abelian Groups of these orders. [3]
   (b) Identify the orders of all of the different elements in your groups and use these to spot any isomorphisms between different ways to write the same group. Explain why some of the different factorisations give the same groups.
2. (a) Choose $K_1 \triangleleft G_1$ and $K_2 \triangleleft G_2$ as non-trivial subgroups of different groups (which are also different from your colleagues' groups). Verify that
   $$(G_1 \times G_2) \,/\, (K_1 \times K_2) \cong (G_1/K_1) \times (G_2/K_2)$$
   (b) Identify a suitable homomorphism to prove this relation in general using the First Isomorphism Theorem.
3. Find an example such that $K \triangleleft H$ and $H \triangleleft G$ but $K \not\triangleleft G$. If you get a different example from your colleagues you will get full marks, otherwise you will have to share them. Try to think creatively, I will give a bonus mark for interesting choices! (register your choice with me and I will let you know if anyone else already claimed those sets)

Math421 Group Theory: Assignment 2b February 2008

Please show all working and reasoning to get full marks for any question.

1. (a) List all of the different integer factorisations of 25, 28 and 31 and hence use the Fundamental Theorem of Abelian Groups to make all of the possible Abelian Groups of these orders. [3]
   (b) Identify the orders of all of the different elements in your groups and use these to spot any isomorphisms between different ways to write the same group. Explain why some of the different factorisations give the same groups.
2. (a) Choose $K_1 \triangleleft G_1$ and $K_2 \triangleleft G_2$ as non-trivial subgroups of different groups (which are also different from your colleagues' groups). Verify that
   $$(G_1 \times G_2) \,/\, (K_1 \times K_2) \cong (G_1/K_1) \times (G_2/K_2)$$
   (b) Identify a suitable homomorphism to prove this relation in general using the First Isomorphism Theorem.
3. Find an example such that $K \triangleleft H$ and $H \triangleleft G$ but $K \not\triangleleft G$. If you get a different example from your colleagues you will get full marks, otherwise you will have to share them. Try to think creatively, I will give a bonus mark for interesting choices! (register your choice with me and I will let you know if anyone else already claimed those sets)

Math421 Group Theory: Assignment 2c February 2008

Please show all working and reasoning to get full marks for any question.

1. (a) List all of the different integer factorisations of 24, 32 and 37 and hence use the Fundamental Theorem of Abelian Groups to make all of the possible Abelian Groups of these orders. [3]
   (b) Identify the orders of all of the different elements in your groups and use these to spot any isomorphisms between different ways to write the same group. Explain why some of the different factorisations give the same groups.
2. (a) Choose $K_1 \triangleleft G_1$ and $K_2 \triangleleft G_2$ as non-trivial subgroups of different groups (which are also different from your colleagues' groups). Verify that
   $$(G_1 \times G_2) \,/\, (K_1 \times K_2) \cong (G_1/K_1) \times (G_2/K_2)$$
   (b) Identify a suitable homomorphism to prove this relation in general using the First Isomorphism Theorem.
3. Find an example such that $K \triangleleft H$ and $H \triangleleft G$ but $K \not\triangleleft G$. If you get a different example from your colleagues you will get full marks, otherwise you will have to share them. Try to think creatively, I will give a bonus mark for interesting choices! (register your choice with me and I will let you know if anyone else already claimed those sets)

Math421 Group Theory: Assignment 2d February 2008

Please show all working and reasoning to get full marks for any question.

1. (a) List all of the different integer factorisations of 29, 30 and 36 and hence use the Fundamental Theorem of Abelian Groups to make all of the possible Abelian Groups of these orders. [3]
   (b) Identify the orders of all of the different elements in your groups and use these to spot any isomorphisms between different ways to write the same group. Explain why some of the different factorisations give the same groups.
2. (a) Choose $K_1 \triangleleft G_1$ and $K_2 \triangleleft G_2$ as non-trivial subgroups of different groups (which are also different from your colleagues' groups). Verify that
   $$(G_1 \times G_2) \,/\, (K_1 \times K_2) \cong (G_1/K_1) \times (G_2/K_2)$$
   (b) Identify a suitable homomorphism to prove this relation in general using the First Isomorphism Theorem.
3. Find an example such that $K \triangleleft H$ and $H \triangleleft G$ but $K \not\triangleleft G$. If you get a different example from your colleagues you will get full marks, otherwise you will have to share them. Try to think creatively, I will give a bonus mark for interesting choices! (register your choice with me and I will let you know if anyone else already claimed those sets)