

Math421 Group Theory: Assignment 1 January 2008

Please show all working and reasoning to get full marks for any question.

1. Let G be the general linear group of degree 2, the set of all 2×2 matrices with non-zero determinant under matrix multiplication and let S be the special linear group of degree 2, that is the subgroup of those which have determinant 1.
 - (a) Find all matrices which are in the centre of G . Is it also the centre of S ? [5]
 - (b) What is the centraliser of your registration number matrix? ($\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ from 20xxabcd).
Verify that this set of matrices contains the centre of G . [3]
 - (c) Determine all elements in S of order 2 and one of order 4. [3]
 - (d) Find a subgroup of G generated by two elements of order 2 which has infinite order, and two other subgroups which are isomorphic to C_4 and V , the Klein 4-group. [6]
 - (e) In general, if $H \leq G$, does the centre of H have to be a subset of the centre of G ? [2]
2.
 - (a) Create the group table for D_6 which is the set of rotations and flips of a regular hexagon. Use a for the rotation of sixty degrees and b for the flip about the vertical axis. [3]
 - (b) Logically identify all subgroups in the group and determine which are normal. [5]
 - (c) Find the conjugates of one of the non-normal subgroups. [2]
 - (d) Is it true that in general any two conjugates have only the identity in common if they are not identical? [3]