

Math421 Group Theory: Assignment 4 April 2006

Please show all working and reasoning to get full marks for any question.

1. The octahedron is the regular solid formed by taking two square based pyramids and placing their two bases together.
 - (a) List the rotational symmetries of the octahedron and describe their effects on the faces, edges and vertices.
 - (b) Determine the number of ways in which an octahedron can have:
 - i. 4 faces selected.
 - ii. 1 vertex and 2 edges selected
 - iii. all edges coloured with one of n colours
 - iv. all vertices coloured with one of n colours
 - (c) Check the n colour vertex count by substituting $n = 2$ and $n = 3$ into your answer and enumerating all of the different ways possible (using symmetry arguments where necessary).
2. (a) Which of these are group actions where g is from some group G ? Check both axioms for all of the candidate actions.
 - i. $g \circ x := -x$ where $x \in \mathbb{Z}$.
 - ii. $g \circ x := \pi$ where $x \in \mathbb{R}$.
 - iii. $g \circ x := g^x$ where $x \in \mathbb{Z}$.
 - iv. $g \circ x := gxg$ where $x \in G$.
3. (a) Find a permutation representation for Q_8 using the method from class.
 - (b) Verify for your representatives of a and b the inverses of all elements and both products of a and b are as expected.
 - (c) Explain why there isn't be a representation for Q_8 which is a subgroup of S_5 .