# Math421 Group Theory: Assignment 3 March 2006

Please show all working and reasoning to get full marks for any question.

1. (a) By finding a generator prove that $\mathbb{Z}_m \times \mathbb{Z}_n$ is actually $\mathbb{Z}_{mn}$ if $m$ and $n$ have least common multiple $mn$.

   (b) Explain why this statement is "if and only if" by considering what happens if there is a common divisor $d > 1$.

   (c) Using this result and the fundamental theorem of finite Abelian groups (that all such groups are cross products of cyclic groups), determine all Abelian groups of orders 47, 48 and 49.

2. (a) Verify that this is a homomorphism from $D_n$ to $V$:

$$\phi(a^{2i}) = p , \quad \phi(a^{2i+1}) = q , \quad \phi(ba^{2i}) = r , \quad \phi(ba^{2i+1}) = s , \quad i \in \mathbb{Z}$$

   (b) What group is the kernel of this homomorphism isomorphic to?

   (c) Verify for the case $n = 8$ that the kernel is a normal subgroup of $D_8$.

   (d) Check the first isomorphism theorem for this particular homomorphism.

   (e) Using the kernel and $< b >$ as your subgroups check that the conditions and the statement of the second isomorphism theorem hold for this case.

   (f) Explain why there are no other normal subgroups of $D_8$ of order 2 or 4 and hence give two other different non-trivial homomorphisms from $D_8$.