Math415 Graph Theory: Assignment 4 (December 2009)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished.

1. (a) Explain why, in the proof that a graph G (with $|V(G)| = n = 10$) is Hamiltonian if $\rho(v) \geq \frac{n}{2}$ for all vertices v, any path P not of length n can be made longer using a vertex outside of P. [2]

   (b) Give an example of a family of graphs with $n > 5$ vertices which are not Hamiltonian but have all vertices of valency at least $\frac{n-2}{2}$. [2]

   (c) Assuming now we have a graph G which has no restriction on its vertices' valencies, but which has a path containing all n vertices between two vertices u and w which are not joined. Prove, using a similar argument to that given in class, that if $\rho(u) + \rho(w) \geq n$ then G is Hamiltonian. [3]

   (d) Explain why a graph G with the condition that any two non-adjacent vertices satisfy $\rho(u) + \rho(w) \geq n$ will be forced to contain a path as in part (c), by using a "longest path" argument. [5]

2. Choose a non-Hamiltonian 2-connected graph J on 10 vertices with maximum valency 4. Show that J^2 is Hamiltonian. Create a tree T with 10 vertices such that T^2 is not Hamiltonian. [4]

3. Find the values of the following parameters for these graph products in terms of the parameters for the graphs G and H. Try with simple examples for G and H to start with, and try to prove your answer in general. [9]

$$\mu(G \times H) \ , \quad \alpha(G + H) \ , \quad \delta(G \circ H)$$