

## Math415 Graph Theory: Assignment 2 (October/November 2007)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and if it is detected you will be punished.

1. (a) Explain why the largest number of vertices possible in an  $r$ -regular graph of diameter  $d$  is [3]

$$\frac{r(r-1)^d - 2}{r-2}$$

- (b) Evaluate this formula for  $d = 1$  and  $d = 2$  and explain what graphs you have met already in the course meet these bounds for certain  $r$ . [3]
- (c) For  $r = d = 3$  it has been proven that no such graph can exist with 22 vertices. Create cubic graphs with diameter 3 which have 12, 14 and 16 vertices. [6]
- (d) What is the best inequality you can prove relating girth and radius? Give examples of graphs which meet the inequality exactly and which differ from it greatly. [3]
2. (a) List logically all trees with 9 vertices and diameter 4 or 6. [6]
- (b) What is the maximum valency in a tree with  $n$  vertices and diameter  $d$ ? [2]
3. (a) Prove that if the minimum valency in an  $n$  vertex graph  $G$  is at least  $\lceil \frac{n-1}{2} \rceil$  then  $G$  is connected. [3]
- (b) Find a family of disconnected graphs which have minimum valency  $\lfloor \frac{n}{2} - 1 \rfloor$  which shows that this characterisation is best possible. [2]
4. (a) What are the centre and circumference of  $G \circ H$  in general? [3]
- (b) Under what circumstances can  $G \circ H$  be a bipartite graph? [2]