Math415 Graph Theory: Assignment 2 2005

Please show all working and reasoning to get full marks for any question.

- 1. Find all trees with 7 vertices by working logically through the diameters.
- 2. Determine the eccentricities of each vertex in $C := G \circ H$ and hence evaluate C's diameter and radius in terms of those of G and H.
- 3. Create different 4-regular graphs, one with each possible pair of edge and vertex connectivities equal to 1, 2 or 3, or explain why such a graph cannot exist. Explain why the connectivity of the given graphs isn't less than claimed in each case.
- 4. Find the minimum and maximum number of edges in a connected bipartite graph on n vertices by investigating the bipartite graphs amongst the graphs with 4, 5 and 6 vertices or otherwise.
- 5. Explain why the adjacency matrix of a simple graph is always symmetric and deduce from your knowledge of linear algebra that all of the eigenvalues of a graph are always real numbers. By giving an eigenvector of any r-regular graph show that r is always an eigenvalue of such a graph.