## Math415 Graph Theory: Assignment 22005

Please show all working and reasoning to get full marks for any question.

1. Find all trees with 7 vertices by working logically through the diameters.
2. Determine the eccentricities of each vertex in $C:=G \circ H$ and hence evaluate $C$ 's diameter and radius in terms of those of $G$ and $H$.
3. Create different 4-regular graphs, one with each possible pair of edge and vertex connectivities equal to 1,2 or 3 , or explain why such a graph cannot exist. Explain why the connectivity of the given graphs isn't less than claimed in each case.
4. Find the minimum and maximum number of edges in a connected bipartite graph on $n$ vertices by investigating the bipartite graphs amongst the graphs with 4,5 and 6 vertices or otherwise.
5. Explain why the adjacency matrix of a simple graph is always symmetric and deduce from your knowledge of linear algebra that all of the eigenvalues of a graph are always real numbers. By giving an eigenvector of any $r$-regular graph show that $r$ is always an eigenvalue of such a graph.
