Math4101 Graph Theory: Assignment 3 (March 2015)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished.

1. (a) Take your graph G from assignment 1 and show it is non-planar by finding a $K_{3,3}$ or K_5 minor within it. [2]

   (b) Use the Euler-Poincaré formula to predict how many faces there will be if G is to be embedded on the torus, and then explain using algebra why it cannot be embedded such that all faces are of size at 3 or 4. Embed G on the torus and/or projective plane in such a way as to make as many of the 3-cycles as possible not faces and to get several different embeddings with different face sizes. Note that some guidance on embeding on the projective plane is found here: `cornellmath.wordpress.com/2007/07/09/graph-minor-theory-part-4/`. [3]

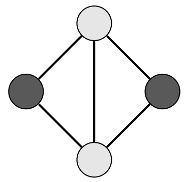
   (c) Find minimal vertex and edge colourings for G, explaining why fewer colours cannot be used. [2]

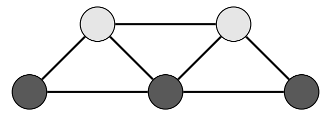
2. Suppose G is a simple planar graph with minimum valency 3 such that $\rho(u)+\rho(v) > 13$ for all edges uv in G. We cannot assume that G is a triangulation, but can assume that we have maximised the number of edges of G while respecting the valency condition.

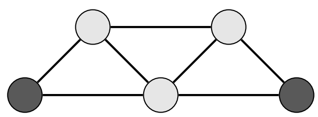
   Assign a charge of $6 - \rho(v)$ to every vertex v in G, verify that the total charge on the graph is positive and discharge by simultaneously distributing all of the charge on each **positively charged** vertex to all of its neighbours. By considering a vertex w which is supposed to have a positive charge after discharging, explain why, whatever the valency of w (starting with the case $\rho(w) \leq 8$), it cannot exist. [6]

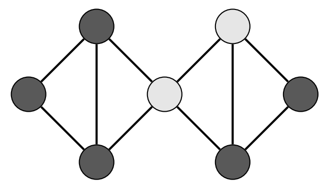
3. (a) Given your configuration (dark is valency 5, light is valency 6), determine its ring size and choose 5 essentially different ring colourings, at least two of which require an argument involving Kempe chains. [7]

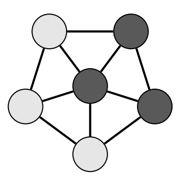
   (b) Suppose that G is a *bipartite* graph with maximum valency Δ. By removing an edge from a subgraph of G, colouring the remaining edges of the graph and using edge chains when necessary, prove that G is edge-colourable with Δ colours. [5]

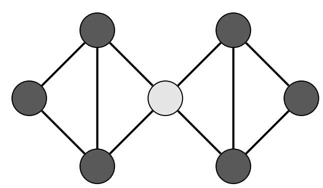
Yurij: 

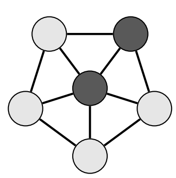
Brittney: 

Matt: 

Emily: 

Mark: 

Kelsey: 

Becky: 

Chris: