Math4101 Graph Theory: Assignment 1a (January 2012)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished.

1. An alternate tool to decide whether or not a valency sequence in non-increasing order $(d_1, d_2, \ldots, d_n)$ is graphical or not is the following:

$$\forall k; \sum_{i=1}^{k} d_i \leq \left(k(k-1) + \sum_{j=k+1}^{n} \min(k, d_j) \right)$$

   (a) Check using Havel-Hakimi that (6,5,5,4,3,1,1,1) is not graphical and find for which k the above formula fails. Find a possible valency sequence for 8 vertices which fails even more often. [4]

   (b) Create a graph G with 8 vertices and 17 edges with $\Delta \leq 6$ and $\delta \geq 2$ which has a different valency sequence from anyone else in the class and see how close the above formula comes to failing. [3]

   (c) Use the edge-switching idea from Havel-Hakimi to create a different graph with the same valency sequence as G, which has a vertex of highest valency with a different sum of its neighbours' valencies to G and notice how the tool works differently this time. [4]

   (d) Draw the complements of both graphs from the previous parts. [2]

   (e) Using evidence from the above parts and by considering the total valency of the subgraph induced by the k vertices of highest valency in a graph G, explain where the terms $k(k-1)$ and $\min(k, d_j)$ come from in the formula and why it holds for any graph G. [4]

2. Find all graphs with valency sequence (5,5,3,3,3,3,2), explaining why none are isomorphic to each other. Make sure that you give all details in words of the steps you took and how you are sure that you missed none. [8]

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2. Find all graphs with valency sequence (4,4,4,4,4,3,3), explaining why none are isomorphic to each other. Make sure that you give all details in words of the steps you took and how you are sure that you missed none. [8]