# Math325 Number Theory: Assignment 1 (September 2010)

Answer all questions and give complete reasons and checks for your answers. Hand in ALL of your rough working together with your final answers. The parts of the questions are weighted as shown on the right of the question. Use of Maple to investigate or check answers is encouraged where appropriate but all working must be given by hand. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

1. (a) Find the prime power decomposition of your number $n$. [2]

   (b) Pick a prime number greater than 300 and different from all others in the class and prove that it is indeed prime. [3]

   (c) Explain why $\gcd(n, p) = 1$ for any such prime $p$ you could have chosen. [3]

   (d) Use the Euclidean algorithm to get $k$ and $l$ such that $kn + lp = 1$. [4]

2. Twin primes are pairs of prime numbers such that their difference is 2.

   (a) Find all numbers which are part of two different twin prime pairs, explaining why none others can exist. [3]

   (b) Use the sieve method to find the next twin primes after $n$. [5]

3. Twin primes are pairs of prime numbers such that their difference is 2.

   (a) Prove that if $a|b$ then $a|b^2$. [2]

   (b) Is it true that if $a|b^2$ and $a < b$ then $a|b$? What if we insist that $a$ is prime? [3]