

Math325 Assignment 4: Legendre, Bases and Continued Fractions

March 31st, 2009

Answer all questions and give complete reasons and checks for your answers. Hand in ALL of your rough working together with your final answers. The parts of the questions are weighted as shown on the right of the question. Use of Maple to investigate or check answers is encouraged where appropriate but all working must be given by hand. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

You should attempt one of the first three questions in this assignment working *solely* in base 12, that will earn you an extra 4 points. The numbers you have been given are in decimal, but you should change them to duodecimal for your chosen question and never change back.

1. Show that the value of your Legendre symbol $\left(\frac{x}{y}\right)$ is 1 by using the rules established in class and then find the root of the associated equation using the powering method. You can assume x and y are both prime numbers. [6]
2. Find the continued fraction form of $\frac{y}{x}$ and z . [6]
3. Determine comprehensive rules for divisibility by 7, 9 and the next two integers after 9 if a number is written in base 8 (octal). [6]
4. Suppose that p and q are odd primes which are not congruent modulo 4 and that a is not a divisor of either p or q . Use the result proved in class:

$$\text{“If } q \equiv \pm p \pmod{4a} \text{ then } \left(\frac{a}{p}\right) = \left(\frac{a}{q}\right)\text{”}$$

to prove that the quadratic reciprocity law holds for p and q . [4]

$$x := 347, \quad y := 607, \quad z := \frac{1 + \sqrt{7}}{3}$$

$$x := 419, \quad y := 547, \quad z := \frac{1 + \sqrt{17}}{2}$$

$$x := 349, \quad y := 587, \quad z := \frac{1 + \sqrt{13}}{2}$$

$$x := 337, \quad y := 593, \quad z := \frac{1 + \sqrt{11}}{3}$$