Math325 Assignment 3b: Functions and Factorisation

March 2nd, 2009

Answer all questions and give complete reasons and checks for your answers. Hand in ALL of your rough working together with your final answers. The parts of the questions are weighted as shown on the right of the question. Use of Maple to investigate or check answers is encouraged where appropriate but all working must be given by hand. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

1. (a) The function $\mu(n)$ is defined as follows:

$$\text{If } n = \prod p_i^{\alpha_i} \text{ then } \quad \mu(n) = \begin{cases} (-1)^{\sum \alpha_i} & \text{if all } \alpha_i \leq 1 \\ 0 & \text{if any } \alpha_i \geq 2 \end{cases}$$

   (b) Evaluate $\mu(k)$, $\phi(k)$ and $\rho(k)$ for $k = 90$ and list all objects to verify your answer. [4]

   (c) Explain why $\mu(n)$ has the multiplicative function property. [3]

   (d) Using a similar method to that used for $a^p - 1$, prove conditions on q necessary for it to be a divisor of $a^p + 1$ for odd primes p and q. Choose six different numbers (in three pairs) for a and p and factorise $a^p + 1$ using this method or otherwise in each case. [10]

2. (a) Find all primitive roots modulo 29 and choose one (different from your classmates, reserve yours with me early!) and make a table of indices with it. [4]

   (b) Use your table of indices to help you solve these equations: [4]

$$x^5 \equiv 8 \bmod 29 \;, \quad y^2 + 19y + 9 \equiv 0 \bmod 261$$