

# Math325 Assignment 1: Divisibility and Primes

January 15, 2009

Answer all questions and give complete reasons and checks for your answers. Hand in ALL of your rough working together with your final answers. The parts of the questions are weighted as shown on the right of the question. Use of Maple to check answers is encouraged where appropriate but all working must be given by hand. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

1. You have selected a prime  $p$  from the set I created; use it for this question.
  - (a) Use the Sieve of Eratosthenes to verify that  $p$  is prime and to find the next 4 primes after  $p$ . Also verify there are no primes between 1328 and 1360. [6]
  - (b) Use the Euclidean Algorithm to get a least linear combination of  $p$  and 2411. [6]
  - (c) Find a number between 2000 and 3000 which requires more steps in the Euclidean Algorithm than 2411 would (just using positive remainders) and try to explain a relation between numbers that would lead to the largest possible number of steps. [6]
2. We are trying to prove that if  $\gcd(e, f) = 1$  then  $\gcd(e \times f, h) = \gcd(e, h) \times \gcd(f, h)$ . [7]
  - (a) Find values of  $e$  and  $f$  which are not relatively prime for which  $\gcd(e \times f, h) \neq \gcd(e, h) \times \gcd(f, h)$  for some  $h$ .
  - (b) Find a different  $h$  for the same  $e$  and  $f$  for which our relation is true.
  - (c) Prove that our statement is true if  $h$  is prime or  $f = 1$ .
  - (d) Hence, or otherwise, prove the statement is true for any positive integers.