

Math 3207 Assignment 3, late February 2016

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. A maximum of $20 - p_y$ marks can be received for this assignment if you hand your work in y days after the deadline, where p_y is the y^{th} prime number.

1. (a) For your given number t , find the prime power factorisation by using trial division, showing all details. [2]
(b) Find the values of $\mu(t)$, $\tau(t)$, $\sigma(t)$ and $\phi(t)$ and list the divisors explicitly. [2]
(c) Complete the proof that ϕ is a multiplicative function by taking an $a \times b$ grid (for some a and b with $\gcd(a, b) = 1$) and start with the number 1 in a corner and choose a diagonal neighbour in a particular direction and put a 2 in it. Repeat this operation (moving in the same direction each time, wrapping around when you hit the edges) and explain why the entire board will eventually be filled and if you cross out all numbers which are not relatively prime to ab they will be in columns or rows which are totally crossed out. [3]
2. (a) Prove that $(\mu \circ \sigma)(n) = n$ for any positive integer n using Möbius inversion. [2]
(b) Show that the values of $(\mu \circ \phi) \circ \sigma$ and $\mu \circ (\phi \circ \sigma)$ are the same for several non-primes unique within the class. Prove that convolution is associative for any number theoretic functions. [5]
3. (a) Under what circumstances could a multiplicative function $f(n)$ have $f(1) \neq 1$? [1]
(b) Using induction, prove that if $S_f(n) := \sum_{d|n} f(d)$ is multiplicative then so is $f(n)$. [5]

$t := 2645047$

$t := 2634949$

$t := 2730727$

$t := 2662931$

$t := 2629849$

$t := 2643413$

$t := 2604577$

$t := 2720591$

$t := 2627111$

$t := 2649991$

$t := 2807081$