Math 3207 Assignment 5, December 2012

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. (a) What rational number has the continued fraction which has the first 5 non-zero digits of your registration number in order? [2]

   (b) Explain why the convergents of any continued fraction have relatively prime numerator and denominator by using the matrix relation or otherwise. [2]

   (c) Prove that $\sqrt{k^2+2k-1}$ has continued fraction $(k; \overline{1, k-1, 1, 2k})$ for any integer $k \geq 2$. Choose a value for k unique within the class and get a rational approximation for $\sqrt{k^2+2k-1}$ which when squared is within 10^{-11} of k^2+2k-1 [8]

2. Recall that a Carmichael number is a *composite* number n such that $a^{n-1} \equiv 1 \pmod{n}$ for all a such that $(a, n) = 1$.

   (a) Prove that no power of 2 can be Carmichael by finding an a which doesn't satisfy the equation in each case. [3]

   (b) Explain why, if n is Carmichael, then if $p|n$ then we must have $(p-1)|(n-1)$. Using this, explain why no Carmichael number can be the product of fewer than three primes. [5]

3. (a) Prove that n is a prime number if and only if $(x+1)^n \equiv x^n + 1 \pmod{n}$. Note that the coefficients are reduced modulo n but the powers aren't. [3]

   (b) Choose a small number b (unique in the class) such that $b^{588} \equiv 1 \pmod{589}$ is *not true* via repeating squaring. [2]