

Mathematics 3205: Fall 2019/20

Instructor: James Preen

Office: A-129A (first floor, A-wing, at the bottom of the ramp)

Office Hours: Before or after class, and/or by appointment

Class meetings: Lectures: Wednesdays at 11:40 and Fridays at 16:00 in B260-C

Textbook : none, but there are some free books linked to on the website

Grading: At least 4 take-home assignments through the term

The outcome from this course should be an understanding of modern combinatorial mathematics. Past tests and exams are available on the course website:

<http://faculty.capebretonu.ca/jpreen/math3205.html>.

The use of electronic devices in class is allowed, boards can be photographed, explanations can be recorded and Maple will be regularly used and worksheets will be available for copying or download. Students are encouraged to ask for clarification on the material both during and after classes. Class will only be cancelled if campus is closed.

Academic regulations concerning penalties for plagiarism and cheating (page 38 of the university calendar) will be rigidly enforced in this course. Differently-abled students will be accommodated using the JKAC. A supplemental exam for the course will not be made available.

Major Topics:

- Counting and Combinatorial Proof
- Generating Functions
- Graph Polynomials
- Codes and Combinatorics of Words

Math3205 Handout 1: Basic Counting

Counting:

The most general cases of counting involve choosing r objects from a pool of n objects under certain conditions. We have formulae for each of these combinations of conditions;

- *Repetition*: whether or not we can draw “the same object” again once it has been drawn or not, as if we are replacing the object back in the pool of n objects each time we pick. Note: if we have multiple objects which are indistinguishable and we aren’t replacing them, we are still in this case, *assuming* we aren’t going to run out of these similar objects.
- *Order*: whether or not we are picking all r objects at once or in a sequence, such as drawing a hand of cards or picking a soccer team by position.

	Repetition Allowed	Repetition Forbidden
Order Important	n^r a sequence of r rolls of a die n is number of faces of the die	${}_n P_r = \frac{n!}{(n-r)!}$ r lotto balls in order n is number of balls
Order Unimportant	$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!(n-1)!}$ r arrows, how many in each segment? n is number of segments	$\binom{n}{r} = {}_n C_r = \frac{n!}{r!(n-r)!}$ hand of r cards n is number of cards

Pigeonhole Principle:

Given a set of p objects and h groups to place them in, if $h < p$ then at least one group has two objects in it. In general, we can say that one group must have at least

$$\left\lceil \frac{p}{h} \right\rceil$$

objects in it where those straight brackets round up the nearest integer.

Note that it can sometimes be useful to round down, in which case the formula is $\left\lfloor \frac{p-1}{h} \right\rfloor + 1$