Math 3205 Assignment 5 December 2019

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. Let the addition table in $\mathbb{Z}_2[\beta]$ where $\beta^3 = \beta + 1$ be as follows:

$$L := \begin{pmatrix} 0 & 1 & \beta & \beta+1 & \beta^2 & \beta^2+1 & \beta^2+\beta & \beta^2+\beta+1 \\ 1 & 0 & \beta+1 & \beta & \beta^2+1 & \beta^2 & \beta^2+\beta+1 & \beta^2+\beta \\ \beta & \beta+1 & 0 & 1 & \beta^2+\beta & \beta^2+\beta+1 & \beta^2 & \beta^2+1 \\ \beta+1 & \beta & 1 & 0 & \beta^2+\beta+1 & \beta^2+\beta & \beta^2+1 & \beta^2 \\ \beta^2 & \beta^2+1 & \beta^2+\beta & \beta^2+\beta+1 & 0 & 1 & \beta & \beta+1 \\ \beta^2+1 & \beta^2 & \beta^2+\beta+1 & \beta^2+\beta & 1 & 0 & \beta+1 & \beta \\ \beta^2+\beta & \beta^2+\beta+1 & \beta^2 & \beta^2+1 & \beta & \beta+1 & 0 & 1 \\ \beta^2+\beta+1 & \beta^2+\beta & \beta^2+1 & \beta^2 & \beta+1 & \beta & 1 & 0 \end{pmatrix}$$

   (a) Count how many latin squares there are within L of each cardinality (and explain why none exist for some cardinalities) from addition of subsets of the elements $\{0, 1, \beta, \beta+1, \beta^2, \beta^2+1, \beta^2+\beta, \beta^2+\beta+1\}$, keeping them in this order. [no need to count all permutations of them] [4]

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α	β	$\beta+1$	β^2	β^2+1	$\beta^2+\beta$

   (b) Using your randomly chosen element α find the 8×8 latin square M which is formed using $s\alpha + t$ for the (s,t) entry using the elements. Do not just fill out the matrix, explain your process as you go along. [4]

   (c) Verify that M is orthogonal to L by highlighting all elements in L which are in the same place as each of the elements of M in a different colour/shape/pattern and verify that the expected behaviour happens which proves orthogonality. [2]

2. (a) Choose a prime number q congruent to 3(mod 4) (different from anyone else in the class you have talked to) and find the squares mod q, use them to form the Paley design P_q for q, and verify that any number appears in the same number of blocks with all other numbers. [2]

   (b) Find the parameters v, b and r for P_q, give the incidence matrix and verify that k and λ satisfy the equations relating the parameters. [3]

   (c) Create the Hadamard Matrix H corresponding to P_q and verify that $H^T H$ is a multiple of the identity matrix, and list the 3-design based on it, verifying how many times a triple occurs. [3]

   (d) Choose a block c of P_q and form either the derived or residual design with respect to c and calculate the parameters. [2]