Math 3205 Assignment 4 November 2019

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. You have selected a graph J with 9 vertices and 11 edges.

   (a) i. Evaluate $i(J)$, the independence polynomial of J, using the iterative relation and the proven formulae for paths and cycles. [3]

   ii. Check that your $i(J)$ is log concave and identify all independent sets of the largest cardinality in J. [2]

   (b) i. Use at least 3 of the different procedures introduced to evaluate the chromatic polynomial of J, $P(J,t)$ and verify that $P(J,2) = 0$ despite J not having any triangle in it. [3]

   ii. Expand $P(J,t)$ and verify that the coefficients of t and t^8 are as expected, and that the coefficient of t^7 is 55. Prove by induction on the number of edges in the graph that the coefficient of t^{n-2} in a graph containing n vertices, m edges and τ triangles is equal to $\binom{m}{2} - \tau$. [4]

2. A matching in a graph is a subset M of its edges such that no two edges in M have any vertices in common. Suppose there are m_j matchings of cardinality j. If G is a graph with n vertices its matchings polynomial can be defined as

$$\mu(G,x) := \sum_{j=0}^{\frac{n}{2}} (-1)^j m_j x^{n-2j}.$$

   (a) Evaluate $\mu(\overline{K_n}, x)$ and find an expression for the m_j in $\mu(K_n, x)$. [1]

   (b) Explain why, for a disconnected graph $H := G_1 \cup G_2$, that [2]

$$\mu(H,x) = \mu(G_1,x) \times \mu(G_2,x).$$

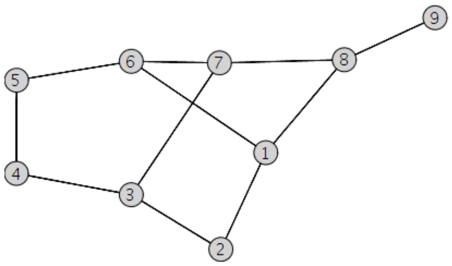
   (c) Prove that for an edge e of G between vertices u and v, [2]

$$\mu(G,x) = \mu(G-e,x) - \mu(G - \{u,v\}, x).$$

   (d) Pick a non-complete symmetrical graph S that nobody else chooses which has at least 6 vertices of valency greater than 2. Use it to demonstrate [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}(\mu(S,x)) = \sum_{w \in V(S)} \mu(S-w,x).$$

   Explain why this is true for any S and hence why the chosen definition used x^{n-2j}.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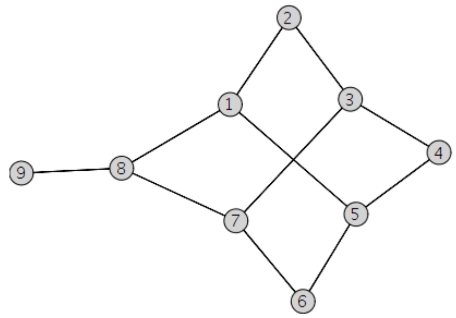
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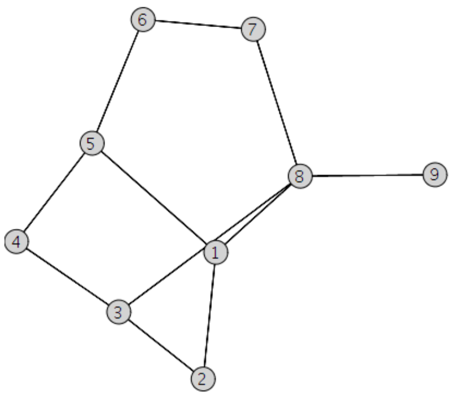
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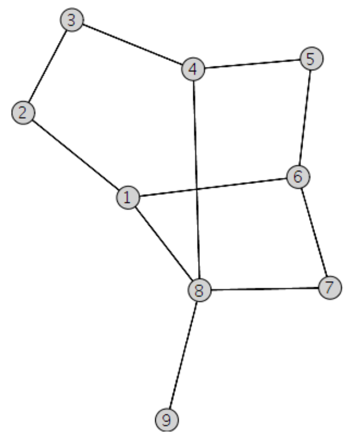
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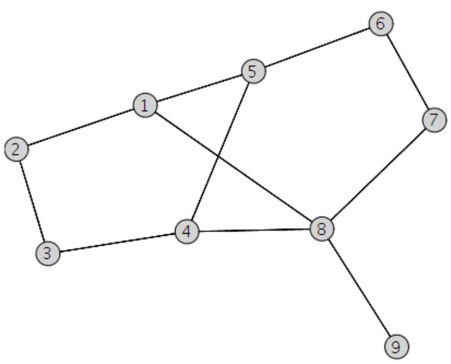
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