Math 3205 Assignment 3 late October 2019

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. (a) Complete the De Bruijn graph with 8 vertices that we started in class. Choose an Eulerian cycle other than the one we used (and different from everyone else in the class) and show that it gives a 19 letter word w containing all 4 letter words as factors. What is the period of w? [3]

   (b) Find the Lyndon factorisation of w, the factoring of $w = l_1 l_2 \dots l_k$ such that $l_j \geq l_{j+1}$ and all l_j are Lyndon, checking these properties. [2]

   (c) Locate a longest factor v of w satisfying $\widetilde{v} = \overline{v}$, explaining why it is longest. [1]

2. The Fibonacci word is the infinite word $F := \lim_{n\to\infty} g_n$ where $g_{n+1} = g_n g_{n-1}$, $g_0 = 1$ and $g_1 = 0$. Its morphism has $\psi(0) = 01$, $\psi(1) = 0$, and $\psi(g_n) = g_{n+1}$.

   We want to prove that, for $n \geq 4$,

   $$P_n :\equiv \text{“}g_{n+2} = g_{n-3} \dots g_1 \widetilde{g_n} \widetilde{g_n} t_n\text{”},$$

   where $t_n = 01$ if n is even and $t_n = 10$ otherwise, and $\widetilde{w}$ is the reversal of w.

   (a) Verify P_4 and P_5. Is P_3 still a true statement? [2]

   (b) Apply the morphism on both sides of our statement P_k to show P_{k+1} is also true, by considering $\psi(t_n)$ and $\psi(\widetilde{g_n})0$. [4]

   (c) Explain why P_n shows that $\widetilde{g_n}$ is a right special factor of F. [1]

   (d) Assume there are two different right special factors of the same length in F and let z be their longest common suffix. Get a contradiction of a property of the F involving factors of the form azb to show there is only one right special factor of each length. [1]

3. (a) Given your real number β, find the first 30 bits of its cutting sequence, and verify that this sequence has the correct number of factors of each length up to 6 to be Sturmian, then show that the Thue-Morse word isn't Sturmian by counting the number of factors in it up to length 6. [3]

   (b) Evaluate β to 5 decimal places, create a Huffman code using the specified frequencies and encode your 5 numbers using it. Repeat this process using a binary arithmetic code and compare the efficiency of the two representations. [2]

   (c) Identify the locations of the errors in your 7-bit Hamming coded message. [1]

$\beta := \frac{\sqrt{3}+2}{6}$, 10011100111010011110010011001001111

$\beta := \frac{\sqrt{2}+1}{4}$, 01101010100101101110100011010110011

$\beta := \frac{\sqrt{7}+1}{6}$, 01100110111001001011010011000110100

$\beta := \frac{\sqrt{8}-1}{3}$, 01100111000011000101110011001010110

$\beta := \frac{\sqrt{11}-1}{4}$, 00011011101001101101011100011100110

letter	freq.	letter	freq.	letter	freq.	letter	freq	letter	freq.
0	$\frac{7}{32}$	1	$\frac{2}{32}$	2	$\frac{6}{32}$	3	$\frac{3}{32}$	4	$\frac{1}{32}$
5	$\frac{2}{32}$	6	$\frac{3}{32}$	7	$\frac{1}{32}$	8	$\frac{2}{32}$	9	$\frac{5}{32}$