Math 3205 Assignment 2 October 2019

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. You have selected a recurrence at random from the list on the second page.

   (a) Determine the values of a_2 and a_3. [1]

   (b) Set up the generating function $A(x) := \sum_{j\geq 0} a_j x^j$ and find a quadratic polynomial $f(x)$ such that $f(x) \times A(x)$ is a simple polynomial without any a_j terms. [1]

   (c) Use partial fractions to express $A(x)$ as a sum of expressions of the form $\frac{\alpha}{1-\beta x}$ for different values of α and β. [3]

   (d) Use part (c) to find an expression for a_k in terms of integer powers and check that a_0, a_1, a_2 and a_3 are correct. [2]

2. (a) Evaluate $g(m,n,r) := \sum_k \binom{n}{k}\binom{m}{r-k}$ for $r = 7$ and a choice (unique within the class) of values of m and n from 3 to 6, explaining what values of k give 0. [1]

   (b) Assuming $r > m$ and m and r are fixed but unknown, use the snake oil method to find the value of the coefficients of $G(x) := \sum_{n\geq 0} g(m,n,r)x^n$. Check your answer agrees with your chosen values in (a). [4]

   (c) If $r < m$, explain what happens in the method used in (b). What can the method do if, instead, $r > m$ but $n = 3m$? [2]

   (d) Investigate $g(m, 3m, 2m)$ by considering products of binomial expansions and so find the binomial coefficient it is equal to in terms of m. [3]

3. (a) Substitute $-x$ for x in the Catalan generating function, and hence get the generating function $B(x)$ for the even Catalans; $B(x) := \sum_{k\geq 0} C_{2k} \times x^k$. [1]

   (b) Evaluate $(B(x))^2$ and hence deduce a combinatorial relation that shows that any Catalan number is related to a sum of products of even Catalan numbers. [2]

$$R1 : a_{n+1} := -1\times a_n + 12\times a_{n-1} + 3\times n\,,\quad a_0 := 1\,,\quad a_1 := 6$$

$$R2 : a_{n+1} := 3\times a_n + 4\times a_{n-1} + 2\times n\,,\quad a_0 := 1\,,\quad a_1 := 7$$

$$R3 : a_{n+1} := 2\times a_n + 8\times a_{n-1} + 2\times n\,,\quad a_0 := 1\,,\quad a_1 := 2$$

$$R4 : a_{n+1} := 2\times a_n + 3\times a_{n-1} + 2\times n\,,\quad a_0 := 1\,,\quad a_1 := 11$$

$$R5 : a_{n+1} := 1\times a_n + 12\times a_{n-1} + 3\times n\,,\quad a_0 := 1\,,\quad a_1 := 5$$