Math 3205 Assignment 4 mid November 2014

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to use Maple to investigate and write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. (a) Create the 7 digit binary number corresponding to the last two digits of your registration number and use it as shown in class (as the seed) to create r, the next 15 random bits following the seed using a primitive polynomial of degree 6 that is not the same as that used by anyone you consulted with. [2]

   (b) Determine the Lyndon factorisation of r using Duval's algorithm, showing all steps. [2]

   (c) Prove that, using the $n \times (2^n - 1)$ Hamming matrix H_n, any bit string with $2^n - 1$ bits is different in at most one place from a bit string in the vector space of solutions to $H_n \underline{v} = \underline{0}$. Find the nearest code word to r from H_4. [3]

2. (a) In the Fibonacci word, prove that there can be no factor w such that both $1w1$ and $0w0$ are factors by assuming w is such a factor of minimal length and then demonstrating that there would then be a smaller factor with the same property. [4]

   (b) Use (a) to explain why there cannot be two different words of the same length such that they both exist as factors followed by both 0 and 1 in the Fibonacci word. eg. if r0 and r1 and t0 and t1 are all factors of the same length then $r = t$. [2]

   (c) Locate the starting point of a palindrome of length at least 4 in the Fibonacci word that nobody else in the class already chose. Use (a) to explain why there are always at most two palindromes of any particular length why there are two when the length is odd. [2]

3. This is a Huffman code:

a = 11	d=101010	e= 01	g=0010	i= 000
l=101011	n=0011	r=10100	s=1011	t= 100

   Your individual sequences are given on page 2.

   (a) Explain why the given code is a prefix-free code and determine the word in English given your sequence that represents your word. [2]

   (b) How many fewer bits are needed than if you used bit strings all of the same size to represent each letter? Create a word (unique in the class) which has a higher compression ratio than your word using this code. [1]

   (c) Using the positive integer grid plot the path formed by your sequence if you start at (0,0) and move right one unit if the next bit is a 0 and up one unit if it is a 1 and find the slope of the line from the origin to the end, the furthest point on the path vertically away from the line and the shortest binary word which does not appear in your sequence and/or r. Now pick an irrational number β unique in the class, draw the line $y = \beta \times x$ and determine the first 30 bits of the sequence defined by this line. Repeat the previous process and discuss how and why these two situations differ. [2]

Yurij	10111001010011001100100110100
Becky	10100011001010011000001110 11
Lindsay	00101010011001100010001101 1
Kelsey	11001100101010000001101 1100
Matt	10010100000110011001010101101
Leanne	10111110101110000010100011011
Mark	10111001010011001100101010110 1
Brittney	10101101111010000110000011001 0
Chris	10101000010111110111000110100
Emily	00110001001010000100011011