Math 3205 Assignment 5, January 2012

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism, you can talk with others in the class but you should submit only your own work. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. Describe the 2-designs which have $k = 2$ and those which have $k = v$. [2]

2. Use the method from class to find five triples of numbers which give all the different possible differences between 1 and 30, and hence give all the blocks of a (31,3,1) 2-design. Your triples should be different from everyone else's in class (if you find this problem too difficult at least try to find a new (25,3,1) or (19,3,1) 2-design). Check that you have the expected number of blocks in your design. [8]

3. Define the matrix M for any design with b blocks and v varieties as the $b \times v$ matrix which contains a 1 in row r and column c if and only if the r^{th} block contains the c^{th} variety.

   (a) By counting the number of 1s in each row and column explain why $M^T M = (r-\lambda)I + \lambda J$ where I is the $v \times v$ identity matrix and J is the $v \times v$ matrix which is all 1s. [3]

   (b) Use row and column operations on $\det(M^T M)$ to get as many zeros as possible so that a sequence of Laplace expansions gives a polynomial in the parameters of the design. [5]

   (c) Using (b) and that $k < v$, explain why $\det(M^T M) \neq 0$ and why therefore we must have $b < v$ too. [3]

4. Create three different 7×7 latin squares which are all mutually orthogonal to each other, and then find a fourth which is not orthogonal to any of the others, explaining why it isn't. [4]