Math 3205 Assignment 4, November 2011

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism, you can talk with others in the class but you should submit only your own work. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. For this question we will be using a code to translate digits into a word:

Digit	1	2	3	4	5	6	7	8	9	0
Code	11101	11100	01	100	1010	000	1111	1011	001	110
Binary	0001	0010	0011	0100	0101	0110	0111	1000	1001	0000

   (a) Explain why the given code is a prefix-free code and hence decode the following sequence which used it: [3]

   1110110111010101110111110001101011100001100111011110111110001001111110101010111011

   (b) Use the frequencies of the digits of your answer to re-encode it using a Huffman code, and count how many bits you save by doing this compared to the word in part (a). [4]

   (c) Now we will be using the Hamming basis where 1000 is transformed to 1110000, 0100 is transformed to 1001100, 0010 is transformed to 0101010 and 0001 is transformed to 1101001. You receive the message on your slip of paper, decode it, correcting any errors that occur, showing all of your working. [5]

2. An infinite word is generated using the following morphism; letting $w_n := \tau^n(0)$ and $w := \lim_{n\to\infty} w_n$:

$$\tau(0) = 01 \quad , \quad \tau(1) = 10$$

   (a) Determine the first 40 bits of w, list the factors of length 2, 3 and 4 that appear in w and determine which of them are right special factors. [4]

   (b) Explain why w_m is of length 2^m and it always has the same number of zeros and ones. Using the morphism prove that if d_j is the j^{th} digit of w then $d_{2k} = 1 - d_k$ and $d_{2k-1} = d_k$ and hence determine the only places in w where a string of the same digit can appear. [5]

   (c) Verify that τ has the inverse property, that is $\overline{\tau(v)} = \tau(\overline{v})$ for any word v, where the line over the word means switching all zeros for ones and ones for zeros. Use this to explain why w_m is of the form $w_{m-1}\overline{w_{m-1}}$ for any $m \geq 1$ and hence deduce for which m it is true that w_m is a palindrome. [5]

   (d) Explain why no word of length less than 5 can contain all factors of length 2 and give a word of length 5 which does. Find as short a word as you can which contains all the different factors of length 3, explaining why you don't believe there can one shorter. [4]

Stéphane: 11010011001100110101101001011011001

Ardell: 01010111100110010010110000110101101

Clay: 11100000011001000111101110011001011

Evan: 01011101000011111010010011001100110