Math 3205 Assignment 3, November 2011

November 23, 2011

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism, you can talk with others in the class but you should submit only your own work. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. (a) For your given graph calculate the Independence and Chromatic Polynomials by using the reduction methods introduced in class. [6]

   (b) List all the independent sets of cardinality greater than 2 in your graph. [2]

   (c) Factorise the chromatic polynomial as far as possible. Supposing that vertex 1 is Red and vertex 5 is Blue, list all of the different sets of colours for the remaining six vertices and explain how this number of sets is related to your chromatic polynomial. [2]

2. A matching in a graph G is a set of edges of G, none of which have any vertices in common. The matchings polynomial counts all of the matchings in a graph and has the following form where $m(G, j)$ is the number of matchings of cardinality j in G:

$$\mu(G) := \sum_{j \geq 0} (-1)^j m(G, j) x^{|V(G)|-2j}$$

   (a) Find all of the matchings consisting of 4 edges in your given graph. [2]

   (b) Explain any three of these results (contact me to let me know the one you don't want to have to prove, it must be different for each of you): [9]

   - $\mu(G) = \mu(G - e) - \mu(G - \{u, v\})$ for any edge $e = (u, v)$
   - $\mu(G \cup H) = \mu(G) \times \mu(H)$
   - $m(P_n, j) = \binom{n-j}{j}$ (for P_n the path graph with n vertices and $n - 1$ edges)
   - $\frac{\mathrm{d}}{\mathrm{d}x}\mu(G) = \sum_{w \in V(G)} \mu(G - w)$

   (c) Use the results from part (b) to calculate the matchings polynomial for your given graph. [4]

$$G_a := \{(1,3),(1,5),(1,7),(2,4),(2,5),(2,6),(2,8),(3,5),(3,6),(4,5),(4,7),(5,6),(6,8)\}$$

$$G_b := \{(1,2),(1,5),(1,6),(1,7),(2,3),(2,4),(3,8),(4,6),(4,8),(5,7),(6,7),(6,8),(7,8)\}$$

$$G_c := \{(1,3),(1,5),(1,6),(1,7),(1,8),(2,3),(2,4),(2,7),(3,7),(4,5),(4,6),(6,7),(6,8)\}$$

$$G_d := \{(1,2),(1,4),(1,5),(1,7),(2,3),(2,4),(2,8),(3,5),(3,7),(4,6),(4,7),(5,6),(7,8)\}$$