Math2101 Handout 2: Logic Formulae 2019

- Basic Definitions

Set Theory	Notation	Definition	Logic	Notation
Complement	$\overline{S}$ (or S^c)	not in	Not	$\sim p$ (or $\neg p$)
Intersection	$S \cap T$	in both	And	$p \wedge q$
Union	$S \cup T$	in either *or* both	Or	$p \vee q$
Disjoint Union	$S \triangle T$	in either *but not* both	Xor	$p \veebar q$
		If p is true then q is true	Implies	$p \rightarrow q$
		If p and q are the same	If and only if	$p \leftrightarrow q$
Universal Set	$\mathcal{U}$	Always true	Tautology	T_0
Empty Set	$\varnothing$	Never true	Absurdity	$(\sim T_0)$

Truth table:

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \veebar q$	$p \rightarrow q$	$p \leftrightarrow q$	T_0	$\sim T_0$
0	0	1	0	0	0	1	1	1	0
0	1	1	0	1	1	1	0	1	0
1	0	0	0	1	1	0	0	1	0
1	1	0	1	1	0	1	1	1	0

- Logic Relationships

Double Negation	$\sim(\sim p)$	$\equiv$	p			
Commutative	$(p \vee q)$	$\equiv$	$(q \vee p)$	$(p \wedge q)$	$\equiv$	$(q \wedge p)$
Associative	$(p \vee q) \vee r$	$\equiv$	$p \vee (q \vee r)$	$(p \wedge q) \wedge r$	$\equiv$	$p \wedge (q \wedge r)$
De Morgan	$\sim(p \vee q)$	$\equiv$	$(\sim p) \wedge (\sim q)$	$\sim(p \wedge q)$	$\equiv$	$(\sim p) \vee (\sim q)$
Distributive	$p \vee (q \wedge r)$	$\equiv$	$(p \vee q) \wedge (p \vee r))$	$p \wedge (q \vee r)$	$\equiv$	$(p \wedge q) \vee (p \wedge r))$
Idempotent	$p \vee p$	$\equiv$	p	$p \wedge p$	$\equiv$	p
Absorbtion	$p \vee (p \wedge q)$	$\equiv$	p	$p \wedge (p \vee q)$	$\equiv$	p
Identity	$p \vee (\sim T_0)$	$\equiv$	p	$p \wedge T_0$	$\equiv$	p
Domination	$p \vee (T_0)$	$\equiv$	T_0	$p \wedge (\sim T_0)$	$\equiv$	$(\sim T_0)$
Inverse	$p \vee (\sim p)$	$\equiv$	T_0	$p \wedge (\sim p)$	$\equiv$	$(\sim T_0)$
Xor	$p \veebar q$	$\equiv$	$(\sim(p \wedge q)) \wedge (p \vee q)$			
		$\equiv$	$((\sim p) \wedge q) \vee (p \wedge (\sim q))$			
Implication	$p \rightarrow q$	$\equiv$	$(\sim p) \vee q$			
Biconditional	$p \leftrightarrow q$	$\equiv$	$((\sim p) \vee q) \wedge (p \vee (\sim q))$			
		$\equiv$	$(p \rightarrow q) \wedge (q \rightarrow p)$			
		$\equiv$	$(\sim(p \vee q)) \vee (p \wedge q)$			

- Quantifiers

There Exists		$\exists$	
For All		$\forall$	
Negation	$\sim(\forall x\ (p(x))$	$\equiv$	$\exists x\ (\sim p(x))$
	$\sim(\exists x\ (p(x))$	$\equiv$	$\forall x\ (\sim p(x))$
Contrapositive	$\forall x\ (p(x) \rightarrow q(x))$	$\equiv$	$\forall x\ ((\sim q(x)) \rightarrow (\sim p(x)))$
Equivalences	$\forall x\ (p(x) \wedge q(x))$	$\equiv$	$(\forall x\ p(x)) \wedge (\forall x\ q(x))$
	$\exists x\ (p(x) \vee q(x))$	$\equiv$	$(\exists x\ p(x)) \vee (\exists x\ q(x))$
Implications	$\exists x\ (p(x) \wedge q(x))$	$\rightarrow$	$(\exists x\ p(x)) \wedge (\exists x\ q(x))$
	$(\forall x\ p(x)) \vee (\forall x\ q(x))$	$\rightarrow$	$\forall x\ (p(x) \vee q(x))$