## Math2101 Handout 1: Set Formulae 2019

- Basic Definitions

| Terminology | Notation | Definition |
| :--- | :---: | :--- |
|  |  |  |
| Complement | $\bar{S}\left(\right.$ or $\left.S^{c}\right)$ | not in $S$ |
| Intersection | $S \cap T$ | in both $S$ and $T$ |
| Union | $S \cup T$ | in either $S$ or $T$ or in both |
| Disjoint Union | $S \triangle T$ | in either $S$ or $T$ but not both |
| Universal Set | $\mathcal{U}$ | all elements |
| Empty Set | $\varnothing$ | no elements |

- Set Relationships

If element $z$ is inside a set $S$ we write $z \in S$, otherwise we write $z \notin S$.
Two sets are equal if every element in one is also in the other, and vice-versa; we write $S=T$.
If a set $S$ is wholly inside of another set $T$ (or equal to it) we write $S \subseteq T$.
The cardinality of a set $S$ is written $|S|$ and is the number of unique elements in $S$.
We define $R:=\{x: f(x)\}$ if $R$ contains all elements $x$ from the universal set for which $f(x)$ is true.

- Set Algebra

The following are the various set relationships we will establish using Venn Diagrams:

| Complementation | $\overline{(\bar{S})}$ | $=$ | $S$ |
| :---: | :---: | :---: | :---: |
| Commutativity | $(S \cup T)$ | $=$ | $(T \cup S)$ |
|  | $(S \cap T)$ | $=$ | $(T \cap S)$ |
| Associativity | $(R \cup S) \cup T$ | $=$ | $R \cup(S \cup T)$ |
|  | $(R \cap S) \cap T$ | $=$ | $R \cap(S \cap T)$ |
| De Morgan | $\overline{(S \cup T)}$ | $=$ | $(\bar{S} \cap \bar{T})$ |
|  | $\overline{(S \cap T)}$ | $=$ | $(\bar{S} \cup \bar{T})$ |
| Distributive | $(R \cup S) \cap T$ | $=$ | $(R \cap T) \cup(S \cap T)$ |
|  | $(R \cap S) \cup T$ | $=$ | $(R \cup T) \cap(S \cup T)$ |
| Idempotent | $(S \cup S)$ | $=$ | $S$ |
|  | $(S \cap S)$ | $=$ | $S$ |
| Absorbtion | $(S \cup T) \cap S$ | $=$ | $S$ |
|  | $(S \cap T) \cup S$ | $=$ | $S$ |
| Identity | $(S \cup \varnothing)$ | $=$ | $S$ |
|  | $(S \cap \mathcal{U})$ | $=$ | $S$ |
| Domination | $(S \cap \varnothing)$ | $=$ | $\varnothing$ |
|  | $(S \cup \mathcal{U})$ | $=$ | $\mathcal{U}$ |
| Inverse | $(S \cup \bar{S})$ | $=$ | $\mathcal{U}$ |
|  | $(S \cap \bar{S})$ | $=$ | $\varnothing$ |

- Inclusion Exclusion

$$
\begin{gathered}
|S \cup T|=|S|+|T|-|S \cap T| \\
|R \cup S \cup T|=|R|+|S|+|T|-|R \cap S|-|R \cap T|-|S \cap T|+|R \cap S \cap T|
\end{gathered}
$$

