Math 2101 (2018/19)
Workshop 5: Relations

"Yes" or "No" is never a sufficient answer in this course, don't forget the because reasons! The questions marked with a * should be left until the end.

1. We are given the following relations between sets $S := \{w, x, y, z\}$ and $T := \{\alpha, \beta, \gamma, \delta\}$.

$$Q := \{(w, \alpha), (x, \beta), (y, \beta), (w, \delta), (z, \gamma), (w, \delta)\} \quad , \quad R := \{(\alpha, x), (\delta, y), (\gamma, x), (\delta, x), (\gamma, z)\}$$

   (a) Identify whether Q or R are everywhere defined, uniquely defined, onto or one-to-one.

   (b) Form the compound relations $Q \circ R$ and $R \circ Q$ and determine whether or not they have the 4 basic properties we investigated from (a).

   (c) Create R^{-1}, Q^{-1} and $R^{-1} \circ Q^{-1}$ and identify its connection to one of the relations from (b).

   (d) *What is the maximum number of ordered pairs in a relation between two sets on 4 elements so that the relation is not everywhere defined?

   (e) *How many different relations are there from a set of c elements to a set of d elements? How many of these are e.d.?

   [start by counting when c is small, and then when d is, then a general argument]

2. Recall that a relation which is both e.d. and u.d. is a function.

   (a) Explain why any function from a set of c elements must have exactly c arrows, and thus if the second set is of a different cardinality than c then it cannot be both onto and one-to-one.

   (b) Give an example of a function F from a set of 4 objects to a different set which has 4 arrows which is neither 1-1 nor onto as well as a function G between the same two sets which is both 1-1 and onto, and note that G^{-1} is also a function.

   (c) Evaluate $F \circ G^{-1}$ and $G^{-1} \circ F$ to see which properties are preserved and which are destroyed when composing.

3. Let us now look at relations from a set $C := \{\text{red, blue, grey, orange, yellow, green, brown}\}$ to itself. Find the ordered pairs in the following relations:

   (a) i. The relation defined by two elements being related if they share no letter in common.
   ii. Now find the relation where related elements have a letter other than e in common.
   iii. The relation where words with the same number of elements are related.
   iv. The relation where w is related to z if w is before z in the dictionary.

   (b) Explain which of these are equivalence relations, by considering why they would be reflexive, symmetric or transitive.

   (c) *Do your answers for (b) depend on the elements of the set you are given? Give examples of different sets which would have different properties, or explain why they don't matter.