Math 2101 (2018/19)
Workshop 4: Counting Problems

Remember to not just give a number as an answer, but a calculation of the formula used. You should also give an explanation of what counting method needed to be used (giving examples if necessary), explaining why in each case repetition is allowed, partially or not, and whether order is important or not.

1. (a) How many registration numers can there be of the form $201qrstu$ where q is not 0 or 9?
   (b) If there are 308 different schools who have had all of these students registered, use the pigeonhole principle and a related idea to determine the smallest and largest number of students that the school with the greatest enrollment could have, theoretically.
   (c) How many two digit sequences qr have a sum of digits greater than 13? List them.
   (d) How many sets of three digits $\{s, t, u\}$ are there
   (e) Which of the sets in (d) have digits that appear more than once?
   (f) How many sets of three digits as in (e) have no 2s?

2. You are given a set of 12 coloured pencils, all different colours.
   (a) How many ways are there for 5 pencils to have been selected as a group?
   (b) One person splits the pencils into sets of three pencils each, then 6 people vote by a secret ballot on their favourite set. How many different collections of votes could there be?
   (c) Three of the pencils are blunt. List logically all the ways that a sequence of 4 pencils could be selected with respect to bluntness, ignoring colour.

3. At a veterinary clinic during one day there were 6 dogs, 3 cats and 2 rabbits waiting to be treated in the clinic. There are 5 different examination rooms and all animals are sent to a room one at a time.
   (a) If you are there with a dog and a cat, how many different positions are there that you could be called into a room with your animals?
   (b) Not differentiating between the animals of the same species, in how many ways could the last two animals waiting to be seen have been? List them
   (c) Why must at least one of the rooms be used by more than one dog? How many times can another room be used for cats?
   (d) In how many sequences can the first three animals which were not dogs have been sent to the rooms?
   (e) How many different orders could there have been for the first four animals? (by species)