Math 2101 (2018/19)
Workshop 3: Proofs, continued

We are going to work more on our proofs this time:

1. Let us call an integer n such that $n = 3k$ where $k \in \mathbb{Z}$, a triad.
   (a) Prove that if a is a triad and b is a triad then $a^2 - 2b$ is also a triad, directly.
   (b) If $p(n) :\equiv$"n is a triad", identify the two separate possible ways for n to be equal to something if $\sim p(n)$ is true for an integer n.
   (c) Use these cases to show that, by contradiction, if c^2 is a triad then c is a triad.
   (d) Prove that if $d \in \mathbb{Z}$ then $d^3 - d$ is a triad, directly, using factorisation or cases.

2. Recall that the *power set* of a set is the set of all subsets of it.
   (a) Pick any set E of containing two different positive numbers and explain why there are 4 different subsets of E.
   (b) Add one new element to E to make F and explain why there are now 8 subsets of F, using whether or not the new element is in the subset to divide the subsets into two cases.
   (c) Using this idea, prove by induction that there are $2^{|S|}$ subsets for any set S.

3. Let x and w be real numbers such that $x > w$.
   (a) Prove by the direct method that if $w > 0$ then $\dfrac{1}{w} > \dfrac{1}{x}$.
   (b) Explain why, if $w < 0$ and $x > 0$, that $\frac{1}{x} > \frac{1}{w}$.
   (c) Use the contrapositive method to show that if $t \leq 0$ then $tx \leq tw$.
   (d) Now taking $w \leq x < 0$, use (c) to show that we again have $\frac{1}{x} > \frac{1}{w}$.
   (e) Express the logic statement "$x^2 > w^2$ when $w < -x$"' as an if–then statement and then prove the resulting statement using the contrapositive method.