Math2101 Assignment 6: 2019 (Late March)

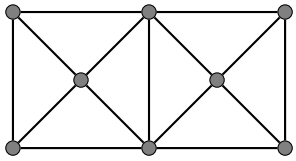
Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

You have selected a piece of paper at random that contains the list of edges for your graph G for the assignment. The vertices of G are $\{a, b, c, d, e, f, g, h\}$.

1. Draw your graph G with the vertices in an alphabetical circle and identify the valencies of the vertices in G. [2]

2. Redraw G as many times as you need to move vertices one at a time to show how you can get a planar drawing of G. Verify that all edges in G are also in your final drawing and all vertices have the correct valencies. [3]

3. Find something structural in G with regards its vertices to explain why G is not isomorphic to this graph H despite having the same valency sequence. [2]



4. Carefully find all the distances from each vertex in G to all of the other vertices to get their eccentricities and hence find the diameter and radius of G. Explain why adding an edge between existing vertices can only decrease the diameter, and adding an edge from an existing vertex to a new vertex can increase the diameter. What is the largest number of eccentricities you can change by doing one of these two operations to G? [4]

5. Demonstrate that H is Hamiltonian by tracing a cycle through all vertices and show that it can be made Eulerian by adding 3 edges and give an edge sequence that passes through each edge once. Are these properties true for G? Show them or explain why not. [4]

6. Carefully construct the complement of G, draw it neatly with as few edges crossing as possible and count how many triples of vertices there are which are mutually joined in $\overline{G}$. [3]

7. Using G and $\overline{G}$ as examples, explain why the vertex connectivity of any non-complete graph is never greater than its minimum valency, but it can be less than that. [2]

$\{af, ag, ah, be, bf, bg, bh, cd, ce, ch, de, dg, eh, fg, gh\}$

$\{af, ag, ah, be, bf, bg, bh, cd, ce, ch, de, dh, eg, fg, gh\}$

$\{af, ag, ah, be, bg, bh, cd, ce, cf, de, df, dh, eh, fg, fh\}$

$\{af, ag, ah, be, bg, bh, cd, cf, ch, de, df, dh, ef, eg, eh\}$

$\{af, ag, ah, be, bg, bh, cd, cf, ch, de, df, ef, eg, eh, fg\}$

$\{af, ag, ah, be, bg, bh, cd, cf, ch, de, df, eg, eh, fg, fh\}$

$\{af, ag, ah, be, bg, bh, cd, cf, ch, de, dg, dh, eg, eh, fg\}$

$\{af, ag, ah, be, bg, bh, cd, cf, ch, de, dh, ef, eg, fg, fh\}$

$\{af, ag, ah, be, bg, bh, ce, cf, ch, de, df, dg, eh, fg, gh\}$