# Math2101 Test 5: 2019 (March $20^{\text{th}}$)

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. We are given the following relations between $A := \{x, y, z\}$ and $B := \{6, 7, 8, 9\}$.

$$R := \{(x,6),(y,8),(x,9),(z,8)\} \quad , \qquad S := \{(7,y),(6,z),(8,x)\}$$

   (a) Identify which ordered pairs in $R$ mean that it is not uniquely defined, why $R$ is not onto and why $S^{-1}$ is a function even though $S$ isn't. [2]

   (b) Form $R \circ S$ and $S \circ R$ as diagrams of arrows between $A$ and $B$ and give one in either set of ordered pairs or tabular form. [2]

   (c) If you need an answer for (b) to do these, just ask me for it in exchange for its mark so you can attempt the following questions.

      i. Draw $R \circ S$ and $S \circ R$ as digraphs on $B$ and $A$. [1]

      ii. Find one arrow which could to be added to $R$ to make $S \circ R$ symmetric. [1]

      iii. Explain why $R \circ S$ cannot be made symmetric by adding any arrows to $R$. [1]

   (d) Explain why there are 4096 different possible relations from $B$ to $A$ by first listing all relations from $\{6\}$ to $A$, and then making a more general argument based on this. [2]

   (e) Create a relation $T$ from $B$ to $A$ such that $T \circ R$ is an equivalence relation, but $T \neq R^{-1}$. Earn a bonus point if you come up with one nobody else does! [1]

2. (a) List or draw the 5 different equivalence relations on the set $\{x, y, z\}$. [2]

   (b) Recall that a permutation is a one-to-one and onto function between a set and itself.

      i. Identify which of the equivalence relations in (a) is also a permutation. [1]

      ii. List or draw all the permutations of $\{x, y, z\}$ and then count how many permutations there will be on a set of cardinality $n$. [2]

      iii. Explain why the composition of two permutations is always a permutation, but show that the composition of two equivalence relations is not necessarily one by giving an example. [2]

   (c) Give two essentially different Hasse diagrams (not permutations of each other) for partial orders on $\{x, y, z\}$ and give each partial order as a set of ordered pairs. Earn a bonus point for counting all possible partial orders on this set! [3]