Math2101 Test 4: 2019 (March 6^{th})

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. In a car park with 7 spaces, there are 7 people who arrive each morning (each driving one car) who want to park there. The cars the people drive are 4 Hondas, 2 Mazdas and a Jeep.

   (a) There are two parking spots that the first two people will usually take, one near the door, the other near the exit. How many different ways are there for the people to have parked in these specific spots on a particular day? (ignoring which car they drive) [2]

   (b) There are 3 parking places under a roof; logically list all of the different sets of types of cars that could be under the roof. Verify your count is correct by identifying which counting formula should be used, with some subtraction. [3]

   (c) We want to investigate the situation in which the order of arrival of the 7 cars (not distinguishing between the cars of the same type) is not repeated for a large number of days, let us say the first day we have [H, H, M, J, H, M, H] for instance.

      i. After 14 days, how many times must at least one of the people have arrived first? [1]

      ii. Calculate the maximum number of days that there can be without a repeat. [3]

      iii. After how many days without repeats must the Jeep have arrived first at least once? [2]

2. We are given 12 different objects, all distinguishable.

   (a) If 3 people (with different names) each select a favourite object without telling the others, then would write their name on the object, how many outcomes can there be? [2]

   (b) Now we are going to split our 12 objects into 3 groups each with 3 objects or more in.

      i. Show there are just three different possible combinations of group sizes. [1]

      ii. Give an example for small n and explain why in general if you split $2n$ objects into two groups of n that there are $\frac{1}{2}\binom{2n}{n}$ different pairs of groups. [1]

      iii. Carefully count all of the different ways the objects could be placed into each of the combinations from i., making sure to count equivalent sets of groups only once. [5] [if you didn't get the three combinations in i., ask me for them to continue this part. if 12 is too large, try a smaller number with a smaller minimum (for some points)]