Math2101 Test 3: 2019 (February 13th)

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. Let $p(n) :\equiv$ " $\sum_{j=1}^{n}(j+5)3^j = \frac{(27+6\times n)\times 3^n - 27}{4}$ "

   (a) Evaluate $p(1)$, $p(2)$ and $p(3)$ to show they are true. Why is $p(0)$ true too? [2]

   (b) Use the method of induction to show that if $p(k)$ is true for some positive integer k then $p(k+1)$ must also be true. [5]

2. (a) Investigate the integers from 16 to 20 as to whether or not they can be represented in the form $(4\times a)+(7\times b)$ for non-negative integers a and b. [2]

   (b) Give a proof by strong induction that all integers greater than 17 can be formed in the way described in (a). [3]

3. (a) Use induction to prove that $n^2 \geq 4\times(n+3)$ for any integer $n \geq 6$. [4]

   (b) Use the contradiction method to show that if $y \geq 6$ then $y^2 \geq 4\times(y+3)$. [4]