Math2101 Test 2: 2019 (January 30th)

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. (a) Plot and/or use algebra on the functions within these logic statements and hence determine which range(s) of real numbers x they are true for. [5]

   $f(x) :\equiv$ "$3x > 2(x+1)$ " , $\quad g(x) :\equiv$ "$x^2 \leq 3(6-x)$ " , $\quad h(x) :\equiv$ "$|x+2| \geq 4$ "

   (b) Give each of the following quantified statements in words (but using $f(x)$, $g(x)$ and $h(x)$) and then explain, in detail, referring to the answers to part (a) or by giving examples, whether each is true or false: [4]

   - $\forall x \in \mathbb{R} : g(x) \vee h(x)$
   - $\exists x \in \mathbb{Z} : f(x) \wedge g(x)$
   - $\forall x \in \mathbb{Z} : h(x) \longrightarrow f(x)$

2. Working step by step, produce the truth tables for both $((p \longrightarrow q) \wedge (q \longrightarrow r))$ and $((p \vee q) \longrightarrow (q \wedge r))$ and show they are logically equivalent.
   (Do *n*ot try to show they are the same using algebra) [3]

3. We want to prove that, for real numbers a and b: if the product of a and b is less than a then a is negative or b is below 1. [5]

   (a) Identify the logical statements in a and b that so that we have $w(a,b) \longrightarrow z(a,b)$.

   (b) Negate $w(a,b)$ and $z(a,b)$.

   (c) Use the contrapositive method to prove that $w(a,b) \longrightarrow z(a,b)$.

4. Use the direct proof method to show that, for integers m and n, if m is odd and $3m-n$ is even then n is odd. [3]