Math 2101 (2016/17)
Workshop 5: Graph Theory

1. A graph G is self-complementary (s-c) if $G^c \cong G$.

   (a) Show that P_4 and C_5 are self-complementary and find the other graph with 5 vertices which is s-c.

   (b) Count the number of edges in an s-c graph with n vertices and hence deduce that the next graph has 8 vertices.

   (c) Create a graph which might be s-c with regards valencies and then check if it is actually is or not.

   (d) Add one vertex and 4 edges to your graph to make a new s-c graph with 9 vertices.

   (e) By joining all vertices from any n vertex graph to its complement, duplicating this graph then joining the two parts carefully show that there exists a s-c graph with $4n$ vertices.

2. The distance between two vertices u and v is the smallest number of edges in a sequence $uv_1v_2 \ldots v$, the eccentricity of a vertex is the largest distance to any other vertex in the graph. A graph has diameter d if all of its vertices have eccentricity d or less. A cubic graph has all vertices of valency 3.

   (a) Show that the diameters of the two non-isomorphic cubic graphs with 6 vertices are different.

   (b) Find two cubic graphs with 8 vertices which are non-isomorphic but which have every vertex of eccentricity 3.

   (c) Determine the unique cubic graph with 10 vertices and diameter 2.

   (d) Explain why there cannot exist a larger cubic graph with diameter 2 by showing that if all vertices are valency k then the maximum number of vertices in such a graph is $k^2 + 1$.

   (e) Try to logically create a graph with 17 vertices of valency 4 with diameter 2, but indicate when things fail for you.