Math 2101 (2016/17)
Workshop 2: Proofs

We are going to investigate the various basic proof techniques in the second lab:

1. (a) Plot these functions using Maple, your calculator or your knowledge of calculus and algebra.

$$f(x) := 2\,x^3 + 11\,x^2 - 26\,x - 35\ , \quad g(x) := |4x + 15|$$

   (b) Let $p(x)$ be the logic statement that $f(x) > 0$ and $q(x)$ be the logic statement that $g(x) \leq 13$. Identify on the real line for which values $p(x)$ and $q(x)$ are true and hence where $\sim p(x)$ and $\sim q(x)$ are true.

   (c) Are these quantified logic statements true or false?: (give reasons)

$$\begin{array}{rcl} \forall x \in \mathbb{R}^+ : q(x) & & (\mathbb{R}^+ = (x > 0)) \\ \exists x \in \mathbb{R} : q(x) & \longrightarrow & p(x) \\ \forall x \in \mathbb{Z} : q(x) & \longrightarrow & p(x) \\ \exists x \in \mathbb{Z} : q(x) & \wedge & (\sim p(x)) \end{array}$$

   (d) Create an algebra based logic statement $r(x)$ which isn't always true such that $p(x) \vee r(x)$ and $q(x) \vee r(x)$ are true for all integers x but not for all real numbers.

2. (a) Prove using the contrapositive method that, for integers n, if n^2 is even then n must be even. Think about why the direct method will be difficult to use here.

   (b) Now set up the contradiction method for the statement that if $m^2 = 2$ then $m \neq \frac{a}{b}$ for any integers a and b.

   (c) Use part (a) to prove (b) by assuming that a and b have no integer factors in common and then contradicting this.

   (d) How do (b) and (a) change if you want to show that $\sqrt{3}$ is not a rational number?

   (e) What goes wrong if you try to use the same method for $\sqrt{4}$ or $\sqrt{9}$?

   (f) What can you say about whether $\sqrt{\frac{c}{d}}$ is rational in general?