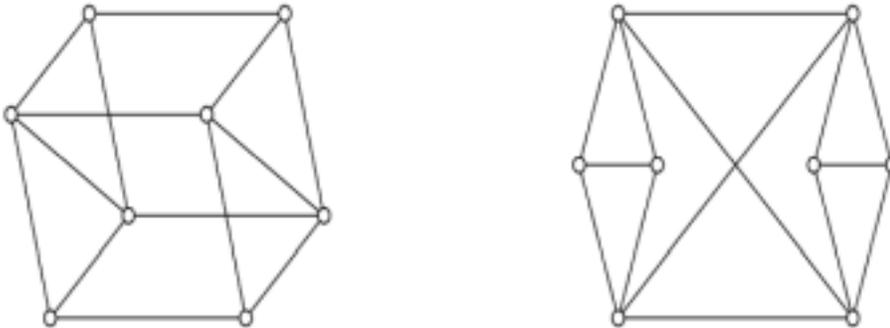


Math2101 Test 5 (Due end of April 2017)

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. You have been given one of these graphs with 8 vertices which are both self-complementary:



- Show that your graph is self-complementary by drawing the complement, labelling it and then re-arranging it to show that it is the same as the original graph. [2]
 - For your graph calculate the eccentricity of each vertex (using symmetry to cut down on the amount of work where possible), the diameter, the chromatic number and the connectedness. Identify one of these parameters that can be used to show the two graphs are non-isomorphic. [4]
 - Find a way to add one vertex and 4 edges to your graph which makes a self-complementary graph with 9 vertices, explaining why. [2]
2. Find all the graphs which have valency sequence $[4,3,3,2,2,2]$ (V) $[3,3,3,3,2,2]$ (K) by using the different ways to remove a vertex of largest valency and then add it back in to the remaining graph. Make sure none of the graphs you finally list are isomorphic to any of the others. [5]
3. The Petersen graph is the unique cubic graph with diameter 2 and 10 vertices.
- Verify that the Petersen graph can have its vertices coloured by 3 colours without any two vertices of the same colour being joined by an edge. [2]
 - Show that the Petersen graph is not a Hamiltonian graph by carefully arguing using its symmetry (any path of 3 edges is similar) but find a Hamiltonian path in your graph from question 1. [3]
4. Prove by contradiction that if a graph has a cut vertex then it is not Hamiltonian. [2]