

30th October 2011

Time : 2-3 hours

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

- Q1.** You have a permutation; a randomly ordered list P of $D := \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
- (a) Let E be the set of numbers which are in the 2nd, 4th, 6th, 8th and 10th positions of P . Let F be the elements of P which are greater than the element to their immediate right in P . Let G be the set of digits whose value is greater than their position in P . Identify the members of E , F and G for your P , giving full reasons, and make a Venn diagram of the three sets. [5]
 - (b) What are the largest and smallest possible cardinalities that E , F or G could have for any permutation of D ? Give an example of the list required in each case as well as explaining why you believe it is the best such one. What permutation can you find that gives the largest value for $|F| + |G|$? [5]
 - (c) Let R be the function from D to itself which is formed by relating the first element of P to 0, the second to 1, third to 2, etc.
 - (i) Find your R and explain why any P will give a 1-1 and onto relation. [2]
 - (ii) Draw the arrow diagram of the relation using just one dot for each digit of D and count how many different cycles there are in this representation of P . [2]
 - (iii) Draw two other cycle patterns different to yours and give the permutations corresponding to these patterns. You will receive a mark for creating valid permutation types that nobody else in the class gives me. (Contact me with your answers to this to reserve yourself as first to use a particular one) [5]
- Q2.**
- (a) Choose two integers, both being odd one being negative. Calculate the difference between their squares and the number one less than their product. Are either integer multiples of 4? [1]
 - (b) Prove, using the direct method, that the difference between the squares of any two odd numbers is always a multiple of 4. [5]
 - (c) Prove by contradiction that if m and n are integers, n is negative and $((m \times n) - 1)$ is negative then m is not negative. Why must m and n be integers? [5]

END OF QUESTION PAPER