Cape Breton University

MATH2101

DISCRETE MATHEMATICS

23rd November 2011 Time : 2-3 hours

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

Q1. (a) Prove by induction that, for any integer $n \geq 1$: [6]

$$A \cup (B_1 \cap B_2 \cap \ldots \cap B_n) = (A \cup B_1) \cap (A \cup B_2) \cap \ldots \cap (A \cup B_n)$$

(b) Verify this relation when $A := \{3, 5\}$ and where B_1 is the set containing the first four digits of your registration number, B_2 contains the third to the sixth digits and B_3 contains the last four digits. Give a registration number different from that given by any of your classmates which would have the largest possible cardinality for the set in (a), explaining why. [4]

Q2. (a) How many registration numers can there be of the form $20pqrstu$ where the date of issue was in the last 8 years? If there are 211 schools who have had at least ten students registered and every possible registration number has been assigned what are the smallest and largest number of students that the school with the greatest enrollment could have? [4]

(b) How many two digit sequences have a sum of digits greater than 13? List them. [2]

(c) How many sets of three digits are there which have no repeated digits? How many such sets have no repeated 1s? [4]

Q3. (a) Create a graph G with six vertices, 10 edges and valencies between 2 and 4. (your graph should be different from all others I receive, please contact me with your list from (b) to ensure you haven't duplicated yours) [2]

(b) Give your graph as a list of vertices and edges and find a drawing with as few edges crossing as possible and emphasizing its symmetry. [2]

(c) Find a longest trail in your graph which doesn't repeat an edge, explaining why there can be none longer. List one cycle of each length you can find in your graph, explaining if any are impossible. [4]

(d) Colour the vertices of your graph with the fewest number of colours possible. [2]

END OF QUESTION PAPER