Math205 Test 3

November 2010

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please do not erase any working and hand in your rough work too.

1. In a music room there are 7 instruments; three are different guitars, there are two different kinds of xylophones, one drum and one violin.

   (a) How many different types of musical duos (groups of two that play together) can be made using this equipment, just considering the four types of instruments? [2]

   (b) If the wish is now to form a group with 3 different types of instrument, but each of the guitars and xylophones make distinct noises, how many different groups of three instruments can there be? [2]

   (c) List all possible ways that you counted for either (a) or (b). [1]

   (d) When people arrive in the room they will choose one instrument not already chosen. How many people could show up and there be a duplication of instruments? If 5 people show up, how many guitars must be chosen? [2]

   (e) One virtuoso musician wants to use the room to perform 3 different pieces in a sequence while using a different one of the 7 instruments each time. How many different ways are there to accomplish this, just considering the type of instrument? [4]

2. We are given the following relations:

$$R := \{(a, y), (b, w), (c, x), (c, z)\}\ , \quad E := \{(w, c), (x, a), (z, b)\}$$

   (a) Which of R and E are uniquely defined and/or onto? [2]

   (b) Find $E \circ R$ in set form and explain why it is an onto relation. [2]

   (c) Suppose $P : T \to U$ and $Q : S \to T$ are relations between some arbitrary sets.

      i. Explain why $P \circ Q$ cannot be everywhere defined if Q isn't everywhere defined but $P \circ Q$ may or may not be everywhere defined if Q is but P isn't. [2]

      ii. Give an example of a P and a Q which are both everywhere defined but for which $P \circ Q$ is not onto. Try to find the smallest and largest number of possible relations in such a $P \circ Q$ (using $|S|$, $|T|$ and/or $|U|$). [3]