Math 205 Assignment 4a, November 2010

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. Consider your first name followed by your last name without any spaces or accents as W. Rearrange the letters of W into alphabetical order A and set up a relation R from the letters of W to the corresponding letter in order of A. To explain further, say, W is "preen" then A would be "eenpr" so we would get the relation $\{(p, e), (r, e), (e, n), (e, p), (n, r)\}$.

   (a) Draw your relation's digraph and table, and determine the arrows in $R \circ R$. [3]

   (b) What conditions on the initial names for a relation formed in this way could lead to R to being a permutation? [2]

   (c) Count the number of different anagrams of W, carefully taking into account all repeated letters. [2]

   (d) Create an equivalence relation E on the same set of letters which has exactly three different equivalence classes. Is $E \circ R$ an equivalence relation? Could it ever be? [3]

2. Consider the graph G whose edges are:

$$\{ab, ac, ad, bd, bf, ce, cg, de, df, ef, eg, fg\}$$

   (a) Create a graph with the same valencies as G which is non-isomorphic to it, explaining why. [2]

   (b) How few colours are needed to properly colour G? How few edges need to be added or removed to make a graph with colouring number 3? [2]

   (c) How few edges are needed to be added or removed to make G Eulerian? Hamiltonian? [2]

3. Using the methods explained in class, do the following research:

   (a) Given this paper's title and authors, find the journal it was published in, download the paper and identify the publication volume, pages and date. [2]

   "Decomposing a planar graph of girth 5 into an independent set and a forest, Ken-ichi Kawarabayashia and Carsten Thomassen"

   (b) Verify which of the authors is the first named author of the paper referenced as [10] in your paper and find that paper in MathSciNet, giving its MR number. What is its primary classification code? [2]

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