Math205 Test 4

November 25$^{\text{th}}$, 2009

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Start a fresh side of paper for each question.

1. (a) Draw this graph and answer the following questions about it: [8]

$$\{ac, ad, ag, ah, bd, be, bh, bj, cj, de, dh, ef, ei, fi, gj, hj\}$$

   i. What is the connectedness of the graph?
   ii. List an Eulerian circuit in the graph.
   iii. What is the graph's colouring number?

   (b) Create a graph with the same valency sequence which has colouring number 5. [2]

2. You are given two sets, $A := \{c, d, e, f\}$ and $B := \{1, 3, 5, 7, 9\}$. This relation is defined from A to B:
$$R := \{(c, 7), (c, 5), (f, 9), (c, 1), (e, 3)\}$$

   (a) Determine whether or not R is one to one, onto, uniquely defined or everywhere defined. [3]

   (b) Explain why there cannot be a relation from A to B which is uniquely defined and onto, but give such a relation from B to A. [2]

   (c) Evaluate $R \circ R^{-1}$ and determine if it is reflexive, symmetric or transitive or not. [3]

   (d) Give some necessary circumstances and examples of relations S for which $S \circ S^{-1} = S^{-1} \circ S$. [2]