Math205 Handout 1: Sets and Logic Formulae 2009

- Basic Definitions

Set Theory	Notation	Definition
Complement	A^c (or $\overline{A}$)	not in
Intersection	$A \cap B$	in both
Union	$A \cup B$	in either *or* both
Disjoint Union	$A \triangle B$	in either *but not* both
Universal Set	$\mathcal{U}$	all elements
Empty Set	$\varnothing$	no elements

- Set Relationships

  Two sets are equal if they have exactly the same elements in, we write $X = Y$.

  If element is inside a set we write $z \in X$, otherwise we write $z \notin X$.

  If a set X is wholly inside of another set Y we write $X \subseteq Y$.

  The cardinality of a set X is written $|X|$ and is the number of unique elements in X.

  A set is designated $\{x \mid f(x)\}$ if it is made up of the universal elements x for which $f(x)$ is true.

- Set Algebra

  The following are the various set relationships we established using Venn Diagrams:

Complementation	$(X^c)^c$	$=$	X
Commutativity	$(X \cup Y)$	$=$	$Y \cup X$
	$(X \cap Y)$	$=$	$Y \cap X$
Associativity	$(X \cup Y) \cup Z$	$=$	$X \cup (Y \cup Z)$
	$(X \cap Y) \cap Z$	$=$	$X \cap (Y \cap Z)$
De Morgan	$(X \cup Y)^c$	$=$	$(X^c \cap Y^c)$
	$(X \cap Y)^c$	$=$	$(X^c \cup Y^c)$
Distributive	$(X \cup Y) \cap Z$	$=$	$(X \cap Z) \cup (Y \cap Z)$
	$(X \cap Y) \cup Z$	$=$	$(X \cup Z) \cap (Y \cup Z)$
Idempotent	$(X \cup X)$	$=$	X
	$(X \cap X)$	$=$	X
Absorbtion	$(X \cup Y) \cap X$	$=$	X
	$(X \cap Y) \cup X$	$=$	X
Identity	$(X \cup \varnothing)$	$=$	X
	$(X \cap \mathcal{U})$	$=$	X
Domination	$(X \cap \varnothing)$	$=$	$\varnothing$
	$(X \cup \mathcal{U})$	$=$	$\mathcal{U}$
Inverse	$(X \cup X^c)$	$=$	$\mathcal{U}$
	$(X \cap X^c)$	$=$	$\varnothing$

- Inclusion Exclusion

$$|X \cup Y| = |X| + |Y| - |X \cap Y|$$

$$|X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z|$$

- Basic Definitions

Set Theory	Notation	Definition	Logic	Notation
Complement	A^c	not in	Not	$\sim p$
Intersection	$A \cap B$	in both	And	$p \wedge q$
Union	$A \cup B$	in either *or* both	Or	$p \vee q$
Disjoint Union	$A \triangle B$	in either *but not* both	Xor	$p \veebar q$
		If p is true then q is true	Implies	$p \rightarrow q$
		If p and q are the same	If and only if	$p \leftrightarrow q$
Universal Set	$\mathcal{U}$	Always true	Tautology	T_0
Empty Set	$\varnothing$	Never true	Absurdity	$(\sim T_0)$

Truth table:

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \veebar q$	$p \rightarrow q$	$p \leftrightarrow q$	T_0
0	0	1	0	0	0	1	1	1
0	1	1	0	1	1	1	0	1
1	0	0	0	1	1	0	0	1
1	1	0	1	1	0	1	1	1

- Logic Relationships

Double Negation	$\sim (\sim p)$	$\equiv$	p			
Commutative	$(p \vee q)$	$\equiv$	$(q \vee p)$	$(p \wedge q)$	$\equiv$	$(q \wedge p)$
Associative	$(p \vee q) \vee r$	$\equiv$	$p \vee (q \vee r)$	$(p \wedge q) \wedge r$	$\equiv$	$p \wedge (q \wedge r)$
De Morgan	$\sim (p \vee q)$	$\equiv$	$(\sim p) \wedge (\sim q)$	$\sim (p \wedge q)$	$\equiv$	$(\sim p) \vee (\sim q)$
Distributive	$p \vee (q \wedge r)$	$\equiv$	$(p \vee q) \wedge (p \vee r))$	$p \wedge (q \vee r)$	$\equiv$	$(p \wedge q) \vee (p \wedge r))$
Idempotent	$p \vee p$	$\equiv$	p	$p \wedge p$	$\equiv$	p
Absorbtion	$p \vee (p \wedge q)$	$\equiv$	p	$p \wedge (p \vee q)$	$\equiv$	p
Identity	$p \vee (\sim T_0)$	$\equiv$	p	$p \wedge T_0$	$\equiv$	p
Domination	$p \vee (T_0)$	$\equiv$	T_0	$p \wedge (\sim T_0)$	$\equiv$	$(\sim T_0)$
Inverse	$p \vee (\sim p)$	$\equiv$	T_0	$p \wedge (\sim p)$	$\equiv$	$(\sim T_0)$
Xor	$p \veebar q$	$\equiv$	$(\sim (p \wedge q)) \wedge (p \vee q)$			
		$\equiv$	$((\sim p) \wedge q) \vee (p \wedge (\sim q))$			
Implication	$p \rightarrow q$	$\equiv$	$(\sim p) \vee q$			
Biconditional	$p \leftrightarrow q$	$\equiv$	$((\sim p) \vee q) \wedge (p \vee (\sim q))$			
		$\equiv$	$(p \rightarrow q) \wedge (q \rightarrow p)$			
		$\equiv$	$(\sim (p \vee q)) \vee (p \wedge q)$			

- Quantifiers

There Exists	$\exists$		
For All	$\forall$		
Negation	$\sim (\forall x\ (p(x))$	$\equiv$	$\exists x\ (\sim p(x))$
	$\sim (\exists x\ (p(x))$	$\equiv$	$\forall x\ (\sim p(x))$
Contrapositive	$\forall x\ (p(x) \rightarrow q(x))$	$\equiv$	$\forall x\ ((\sim q(x)) \rightarrow (\sim p(x)))$
Equivalences	$\forall x\ (p(x) \wedge q(x))$	$\equiv$	$(\forall x\ p(x)) \wedge (\forall x\ q(x))$
	$\exists x\ (p(x) \vee q(x))$	$\equiv$	$(\exists x\ p(x)) \vee (\exists x\ q(x))$
Implications	$\exists x\ (p(x) \wedge q(x))$	$\rightarrow$	$(\exists x\ p(x)) \wedge (\exists x\ q(x))$
	$(\forall x\ p(x)) \vee (\forall x\ q(x))$	$\rightarrow$	$\forall x\ (p(x) \vee q(x))$