Cape Breton University

Math205

Discrete Math

November 2008

Time : 1.5 hours

Answer THREE of the FOUR questions, giving all working and reasoning.

Q1. (a) Simplify this logic expression to one with just two letters in: [7]

$$((\sim q) \to (r \wedge p)) \to p$$

(b) Verify your result with a truth table. [3]

(c) Give an expression involving p, q and r that simplifies to q. [1]

Q2. (a) Prove, using the direct method, that the difference between the squares of two odd numbers must be an integer multiple of 4. [8]

(b) Explain why this fact is also true for even numbers, but not if one is odd and the other is even. [3]

Q3. (a) Given this universal set of country codes, explain which are in these three sets and use this information to form a Venn diagram. [4]

$\mathcal{U}$:= { NZ, CAN, GB, ZW, IT, PRC, MOR, DDR }
A := Codes with fewer than three letters in
B := Codes which do not include a vowel
C := Codes with more letters from the 2nd half of the alphabet than the first

(b) Which subset(s) of your diagram are empty? Identify one of cardinality 3. [2]

(c) Prove, using Venn diagrams, that $X \cap (Y \cup \overline{Z}) \subseteq \overline{Y} \cup (X \cap Z)$ [5]

Q4. (a) Use algebra to simplify these inequalities and hence plot where they are true on the real line [6]

$$\begin{aligned} p(x) &:\equiv \text{“ } |x-1| \geq 2 \text{ ”} \\ q(x) &:\equiv \text{“ } x^2 < 2 + x \text{ ”} \\ r(x) &:\equiv \text{“ } |3x-1| \leq 5 \text{ ”} \end{aligned}$$

(b) Determine whether these statements are true or false: [5]

$$\exists x \in \mathbb{Z}; p(x) \wedge r(x) \ , \quad \forall x \in \mathbb{R}; p(x) \vee r(x) \ , \quad \exists x \in \mathbb{R}; q(x) \rightarrow r(x)$$

END OF QUESTION PAPER