Cape Breton University

Math205

Discrete Math

December 2008

Time : 3 hours

Answer FIVE of the SEVEN questions, giving all working and reasoning.

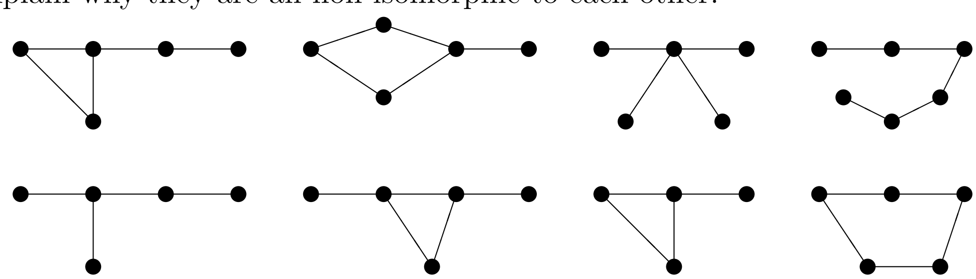
Q1. In a canoeing race, there are 5 Canadians, 2 Germans, 2 Slovakians and a Hungarian. There are no ties and we don't distinguish between the canoeists from the same nation.

(a) How many different ways can the countries finish 1st, 2nd and 3rd? [4]

(b) How many different sets of positions can the Canadians finish in? [3]

(c) At the halfway point a Slovakian is leading the race, but that country does not end up winning. We are interested in how many different ways there are in which the country in every position at the end of the race was different from the country in that position at halfway. List logically all the different patterns possible. [5]

Q2. (a) Determine the valency sequences and colouring numbers of these graphs and hence explain why they are all non-isomorphic to each other. [5]



(b) Categorise the graphs into these three sets and hence make a Venn Diagram: [5]

F := { graphs which have five edges or more }
T := { graphs which require less than three colours }
V := { graphs which have more than one cut-vertex }

(c) Explain why there could never be a graph in $\overline{F} \cap \overline{T} \cap V$. [2]

Q3. (a) Use Venn Diagrams to prove that $X \triangle (Y \cup Z) \subseteq \overline{(X \cap Z)}$. [4]

(b) Prove using the contrapositive method that $|A \cap B| < |A|$ if $B \subset A$. [5]

(c) Use the inclusion-exclusion formula to deduce a similar relation between $|B|$ and $|A \cup B|$. [3]

Q4. (a) Explain why this relation $R := \{(a,a),(b,b),(c,c),(d,d),(a,c),(d,a),(c,d),(a,d)\}$ is neither a partial order nor an equivalence relation. [3]

(b) What is the fewest number of pairs whose removal would turn R into a partial order? How few are needed to be added to make R an equivalence relation? [2]

(c) Evaluate $R \circ R$ and explain why it is an equivalence relation. [2]

(d) Explain why, if S is any partial order, then $S \circ S = S$. [5]

Q5. (a) Prove by induction that the difference between a positive integer and four times its cube is an integer multiple of 3. [7]

(b) Determine at which places on the real line these two logic statements are true and explain why $\forall x \in \mathbb{Z}\ \ (r(x) \rightarrow q(x))$ is false. [5]

$$q(x) :\equiv \text{``} \ |x^2 - 3| > 1 \ \text{''}\ , \quad r(x) :\equiv \text{``} \ x^3 \geq -1 \ \text{''}$$

Q6. (a) Draw the graph $G := \{ab, ae, af, bc, cd, cg, df, ef, eg\}$ and hence determine why it is not Hamiltonian and what is its colouring number. [4]

(b) Create the connected graph with five vertices which needs 4 colours but which isn't Hamiltonian, and explain why this couldn't work with fewer vertices. [3]

(c) Using the pigeonhole principle, prove that every graph (with more than one vertex) must have two vertices of the same valency. Give an example of why this isn't true if we allowed two edges in a graph between the same two vertices. [5]

Q7. (a) Simplify this logic expression to one of each letter and one pair of brackets. [6]

$$(r \rightarrow p) \rightarrow (r \rightarrow q)$$

(b) Using truth tables prove that $\leftrightarrow$ satisfies all three equivalence relation properties for logic statements. [6]

END OF QUESTION PAPER