

November 2008

Time : 2 hours

Answer ALL of the THREE questions, giving all working and reasoning. Plagiarism is not allowed, all work handed in must be your own. Please also include all rough working.

Q1. In a basket there are 3 red apples, 4 yellow apples, 1 red banana, 6 yellow bananas, 4 red grapefruit, 2 yellow grapefruit and 3 pineapples (which are all yellow).

- (a) If 15 people each take one item of fruit, at least how many must have the same colour of some particular fruit? How many people are actually needed for us? [5]
- (b) How many people would there have to be taking an item of fruit to guarantee that 4 yellow bananas have been picked? [2]
- (c) How many ways are there to make a line of 10 colours using these fruits? [3]
- (d) If 4 fruits are picked (ignoring colour) how many ways are there to do this? Logically list all the ways they can be made. [5]

Q2. (a) List logically all graphs with 5 vertices which have fewer than 6 edges. [6]

(b) Breaking the graphs into sets with the same number of edges, how many must there be in one of these sets according to the pigeonhole principle? How many such sets exceed this limit in reality? [3]

(c) Classify all of your graphs from (a) as to whether: they are connected or they are Eulerian or both. [4]

Q3. We define the sets $A := \{a, b, c, d\}$ and $N := \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and these relations:

$$R := \{(c, 1), (a, 9), (d, 7), (b, 3), (d, 2), (c, 6)\} S := \{(b, 5), (a, 4), (c, 5)\}$$

Let T be the relation from A in alphabetical order to the last 4 digits of your registration number in that order.

- (a) Explain whether or not these three relations are 1-1 or uniquely defined. [4]
- (b) Draw or tabulate the functions $R \cup S$, S^{-1} and $S^{-1} \circ T$. [5]
- (c) Count the number of different finite functions that can exist from A to N . [3]

END OF QUESTION PAPER