Math205 Test 1

September 27th, 2007

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown and the questions can be answered in any order. Please start a fresh side of paper for each question.

1. Given the universal set $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$.

   (a) Identify the elements of the following sets and plot them on a Venn diagram: [5]

   $$\begin{aligned} A &:= \{4n + 1 \ ; \ \ n \in \mathbb{Z}\} \\ B &:= \{m^2 - 4 \ ; \ \ m \in \mathbb{Z}\} \\ C &:= \{k \ ; \ \ |2k + 1| \geq 6\} \end{aligned}$$

   (b) Identify which regions in your Venn diagram have the smallest and largest cardinalities. (In the form $X \cap Y \cap Z$ where X is A or $\overline{A}$, Y is B or $\overline{B}$ and Z is C or $\overline{C}$) [2]

   (c) Use your diagram to identify the elements in $(A \cup B) \cap \overline{C}$ and check the inclusion-exclusion formula for A and B. [3]

2. Use algebra to simplify this expression and check your answer with Venn diagrams. [10]

$$(A \cap (B \cup (\overline{A \cup B}))) \cap (A \cup \overline{B})$$